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Positron annihilation spectroscopy of clumpy structures 1. Type II young Supernova remnants

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The clumpy structure of the SN 1987A ejecta revealed by numerous IR observations has been described. The simulation of the clump formation in the progenitor star's explosion was performed. The Sedov self-similar solutions of the problem of a strong explosion were used to construct the model spectra of the initial clumps formed due to the Rayleigh-Taylor (RT) instabilities arisen during the metal core collapse prior the supernova explosion. After short time main part of the clumps accumulates mainly the heavy elements, including radioactive isotopes. In this paper, we investigated the properties of the element mixture with radioactive isotopes, especially $^{44}_{22}Ti$. Mapping of the ejecta clumps was performed through the obtained spectra of their radiation field depending on time, initial spatial distribution and composition. Numerical simulation of positron trapping across nebular envelope supported the new possibilities for positron γ-spectroscopy and chemical diagnostics. The exact results of the quantum annihilation processes with inner K–electrons produce one γ–quant process are tested for SN 1987A envelope clamps. Aspherical clumps distribution presented and used how to test the elements separation after explosion. We presented the accompanied nonthermal emission after positrons energetic losses in cool explosion of the envelope and the distribution of Auger electron nebular plasma with small dust fraction. This secondary emission of atomic quants and fast electrons is connected with aroused diffuse nebular emissions. We found that only one γ–quant annihilation line for chemical elements from the middle part of periodic table of chemical elements should be usable for future positron spectroscopy activity. For exact solution of this problem we selected clamps without graphite dust, which contain the radioactive isotope $^{44}_{22}Ti$, and show the production of standard positron current in the case of standard chemical composition.

Key words: gas-dust space medium, auger – electrons, supernovae remnants, positron spectroscopy.
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1 Introduction

Infrared observations have been an important source of information about supernova remnants over the last 30 years. In general, they provided an insight into the chemical composition of the dust grains consisting of astronomical silicates and amorphous carbon. Observations of SN 1987A have been the most informative. The distance to the SN 1987 (50 kpc) and its current age (31 years) allowed establishing the onset of dust formation in the debris, its physical properties, as well as to follow further evolution of the remnant. The total mass of dust in the remnant of SN 1987A reaches 0.8 $M_{\odot}$.

However, the most important output of the space IR observations in recent years, including those obtained by the Spitzer Space Telescope, is the detection of the fragmentary (clumpy) structure of the ejecta at early stages in its evolution. The time scale of such clumpiness coincides with the time scale of complete hydrogen recombination in the ejecta. The remnant geometry at the current epoch is being reconstructed from the structure of the shock fronts visible in different spectral ranges. As of today, the lower theoretical limit on the number of clumps N_C in the ejecta is determined to be 100. Varosi & Dwek [6] have proposed a technique of the geometry-based simulation of the supernova clumped ejecta with dust grains being the only factored in component. The N_C values may change with increasing sensitivity of the instruments used. The upper limit on the N_C values can be estimated directly from the computations of the supernova explosion model, in particular, in the last pre-explosion stages when the gas is infalling from the envelope onto the dense stellar core. The RT instabilities, being initiated at this stage, induce formation of clumps, and the core matter is driven toward the overlying zones Müller *et al*., [9], Woosley [8]. Such outer zones are present in all simulations of the Type II supernova explosions. Therefore, the N_C values describe the number of such survived clumps derived from the observations of the young remnant of SN 1987A.

One may distinguish among three main phases which the supernova remnant passes as it expands. Being placed in chronological order, these are non-adiabatic, adiabatic and non-adiabatic phases. The first phase occurs on relatively small time scales when material from the envelope falls in and then rebounds off the core ultimately triggering the explosion and explosive ejecting of the matter. By that time (within the first 100 seconds after the outburst) the ejecta clumps have been already formed due to the RT instabilities. At the current epoch, we have been observing the adiabatic phase of the remnant evolution when the energy of all examined processes is by 16-18 orders of magnitude lower than the total kinetic energy of the ejecta (10^{51} erg). In mechanics, changes in the thermodynamic and kinematic parameters of the remnant are described by the similarity method – the Sedov method, which was applied by Zeldovich and Raizer [14] to solve the problem of strong explosion in a relatively rarefied medium.

Taking into account some constraints on the use of a self-similar solution (for the first and third stages of the remnant evolution), the description of the clump structure when switching from small scales to large scales and vice versa in the adiabatic phase appears to be plausible and convenient. Therefore, when solving the inverse problem on the explosion structure reconstruction from the known parameters of the ejecta clumps at the current epoch of observations, the requirement for compliance with the self-similar approximation places a constraint on its application on the considered time and geometric scales of the explosively ejected matter. When the clumps are expanding, they overlap each other with respect to the line of sight. However, according to modern observations, the factor of such overlapping is rather small. Let us express it in terms of ξ. Thus, the ejecta clumpiness makes it possible to perform model mapping of such clumps with further definition of the observation objectives.

The International Gamma-Ray Astrophysics Laboratory (INTEGRAL) observations have revealed a large amount of radioactive isotope of titanium ${}^{44}_{22}Ti$ Grebenev et al [16] in SN 1987A. The presence of this isotope has two important implications. First, even at low mean thermodynamic temperatures, the average energy in the clumps, which is emitted in the non-thermal part of the spectrum, is considerable due to radioactive decays. With regard to SN 1987A, we observe permanent hard radiation with a half-life of 85 years. Currently, the amount of energy $\boldsymbol{E}$ deposited in the debris by the radioactive decay of titanium isotope is 10^{36} erg. Hard primary radiation, which includes emission of fast positrons, is responsible for the distribution of the radiation field that differs significantly from the thermal one. Secondly, the local electron energy distribution function is formed due emission of the high-energy Auger electrons associated with the ionisation loss of fast positrons. The electron constituent of such ionised gas is responsible for the increased IR fluxes from dust and is involved in the ionisation-recombination balance of the gas, in particular, in excitation of atomic outer shells, which define the composition of the ejecta. Considerable experience in diagnostics of interstellar dust by means of the IR spectroscopy has been gained, making it possible to use the deduced quantitative parameters to distinguish between properties of the condensed swept-up interstellar matter and those of the ejecta's own condensate. It also becomes possible to perform diagnostics of the current state of the ejecta atomic constituent under conditions of varying composition of its clumps due to various radioactive decays, in particular, due to a series of β-transformations ${}^{44}_{22}\mathrm{Ti} \xrightarrow{85\,\mathrm{y}} {}^{44}_{21}\mathrm{Sc} \xrightarrow{6\,\mathrm{h}} {}^{44}_{20}\mathrm{Ca}$ in the matter inside the clumps [17, 18]. Therefore, it is apparent that spectroscopic manifestations of actual changes of the chemical composition should be observed in the supernova remnant at the current epoch of observations.

This paper discusses modelling of mechanical and thermodynamic properties of the ejecta clumps from the space-based observations, as well as reconstruction of the initial state of these clumps prior to the supernova explosion. The results of the model spectra imaging within the constraints of the observed spatial pattern of the geometry of the remnant of SN 1987A will be presented in our further studies.

2 The clumpy structure of the ejecta of SN 1987A in the infrared spectra

The study of the chemical composition of the ejecta of SN 1987A requires observations at a rather high angular resolution. The preliminary mapping of the debris suggests that it can be shaped like either a prolate spheroid or a cylinder. The observations have shown that the shock breakout occurred through the poles along the rotation axis of the progenitor and in the equatorial plane. At present, the blast wave fronts have travelled far from the debris and can be clearly observed across all wavelengths covered by modern space telescopes. The velocity of the blast wave propagation is multi-fold higher than the expansion velocity of the ejected matter. Today, more than 31 years after the explosion, the largest telescopes can finally perform mapping of the debris. On the other hand, calculation of Type II supernova explosion models necessitates factoring in asymmetric elemental distribution within the volume of the ejecta. According to recent IR observations of the remnant of SN 1987A, the ejecta clumpiness becomes apparent.

The emission from silicate and carbon-containing dust grains, which is sufficient to resolve geometric sizes in the adiabatic phase of the remnant expansion, serves as a marker of such clumpy structure. Until recently the clumps were illuminated by hard X-ray and ultraviolet output from three blast-wave fronts, which have already travelled quite far, as well as due to the energy deposited by the radioactive decay of titanium isotope -$^{44}_{22}Ti$ in the innermost regions of the ejecta (Jerkstrand et al [1]). Simultaneously, the dust temperature increased, and the dust grains themselves started emitting infrared radiation. Numerous observations of the dust grains in the ejecta and their interpretation by Dwek & Arendt [10] enable us to reconstruct the morphology of the clumps reproducing the IR spectra and define their quantitative characteristics. To solve this problem, let us perform the preliminary analysis of the clump characteristics. To this end, as in Varosi & Dwek [6], we introduce the volume filling factor of the clumps in the ejecta – $\boldsymbol{f_V}$ and the cross section of individual clumps – $\boldsymbol{x}$. Let ξ be a factor of order unity that compensates for the overlapping of clumps along a given line of sight. We also assume that clumps are of the same size ($r_C = x$). Then, from the constraint $N_C r_C^2 = \xi R^2$ for a given spherical clump with radius $\boldsymbol{r_C}$ we obtain its optical depth – $\boldsymbol{\tau_C}$:

$$\tau_C = \frac{3}{4}\frac{M_d/N_c}{\pi r_C^2}, \qquad (1)$$

where $\boldsymbol{M_d}$ is the total dust mass in the ejecta. Determination of detailed elemental abundances throughout the volume of the ejecta requires that within the investigated spectral range of observations the relevant optical depth is subject to the condition $\tau_C \leq 1$. Combined space-based observations in recent years have shown that the total energy fluxes in the X-ray, UV and IR spectra of SN 1987A tend to decrease; this is due to decreased luminosity at the shock fronts and destruction of interstellar dust concentrated in the vicinity of these leading shock edges. In this case, the energy flux is dominated by the emission from the debris of this supernova. The clumps themselves being expanded become optically thinner, making it easier to observe their inner regions. Under given conditions, the mean free path of a photon through the ejecta *l* can be derived as follows:

$$l = \frac{1}{N_C \pi r_C^2} \qquad (2)$$

The growth rate of the clumps in the inertial reference frame relative to an arbitrary point in the expanding debris is many orders of magnitude lower than the expansion velocity of the ejecta. It should also be noted that in this case the clump overlapping factor ξ is negligible and does not exceed 0.1. Given a uniform distribution of the clumps throughout the ejecta volume, we can estimate the overlapping factor ξ by formulae (3) and (4) presented in the next section.

2.1 Boundary conditions for the problem and imaging of the debris.

The bolometric luminosity of the supernova debris together with the emission at blast-wave surface in across all spectral bands makes up 300 $L_\odot$, of which 63 $L_\odot$ contributed by the X-ray emission at the shock fronts. The main energy input for the remaining luminosity of the remnant of SN 1987A is provided by the radioactive decay of $^{44}_{22}Ti$ nuclei leading to the non-thermal fluorescence of gas in the ejecta, and by the emission from the dust grains. Taking into account the modern data on the remnant geometry, its actual size is R=10^{15} m at the expansion velocity of the ejecta layers within the range of 800-1200 km/sec. We have inferred from the hydrodynamics of the supernova explosion that the remnant mass is concentrated in the zone of $0.1*r$ where $\boldsymbol{r}$ is the current position of the outermost part of the remnant relative to the explosion site (Shklovsky, [13]). Factoring in the clumpiness and non-sphericity of the ejecta zones enables us to estimate the density $\boldsymbol{\rho}$ in an ejecta zone. The estimated values are about ρ = 10^{-17}-10^{-18} $^{kg}/m^3$ or 10^9-10^{10} atoms/m^3 (Jerkstrand, [1]). Under given conditions, the translational kinetic energy of the ejecta zone is 13 orders of magnitude higher than the energy deposited by the radioactive decays occurring in this zone.

However, even with such radioactive decay energy input, there is an efficient mechanism of conversion of the radioactive decay energy into the emitted radiation. Thus, there may be preconditions for the ejecta luminosity to become sufficiently high that it can be mapped or spectroscopically studied. Physical conditions at the ejecta surface are similar to those observed at the shocks. Hence, the atomic spectroscopy of the supernova debris should take into account the interaction between the ejecta and circumstellar gas. Eventually, with such a source of conversion of the energy of the debris translatory motion into thermal chaotic motions, the spectrum of multi-charged ions typical at the shocks appears being overlapped by the emission spectrum produced within the volume of the ejecta (inner diffuse ejecta spectrum). This problem is to be discussed in the next paper though.

3 Reconstruction of the original picture of the SN 1987A explosion

The analysis of observations of this supernova is based on the application of the similarity method which is used to define the relationship between the temperature $\boldsymbol{T(M_r t)}$, density $\boldsymbol{\rho(M_r, t)}$ of the ejecta, the distance from the explosion site $\boldsymbol{r}$ and time scale of the process. In many cases, this approach is the most efficient among others due to its physical simplicity, availability and small number of free physical parameters. Being adhered to the similarity method, we are to consider the results of IR observations at the current epoch and parameterize the structure of the clumps in the debris in the same manner as in the previous section. The state of the clumps can be defined by:

$$\rho(M_r, t) = \rho(M_0, t_0)\left(\frac{t}{t_0}\right)^{-3}, \qquad (3)$$

$$T(M_r, t) = T(M_0, t_0)\left(\frac{t}{t_0}\right)^{-3(1-\gamma)}. \qquad (4)$$

where $M_0 = 20M_{\odot}$; $\gamma = 1{,}25 - 1{,}30$. Each clump has its distinct size defined by its own initial conditions. The similarity relations for these clumps are linearly dependent on $\boldsymbol{r}$ and $\boldsymbol{t/t_0}$. The growth of a clump depends on the time and difference between the pressure inside the clump and in the inter-clump medium. The growth of clumps is proportional to the time elapsed since the explosion and it occurs simultaneously with the overall expansion of the remnant. In the simplest case, the size distribution of clumps may be set taking into account that we know their observed root-mean-square size $\boldsymbol{w(x)dx}$, i.e. the probability that within a given range of sizes $\boldsymbol{dx}$ the following is true:

$$w(x)dx = \frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{x^2}{2\sigma}\right)dx \qquad (5).$$

The dispersion coefficient $\boldsymbol{\sigma}$ and mean size of a clump $\boldsymbol{\bar{x}}$ in the above formula should be estimated and compared with the observed values. As of today, we assume without loss of generality that the upper limit on the dispersion $\boldsymbol{\sigma}$ is $\sigma = 10^{13} - 10^{14}$m. To get a complete picture of the explosion, it is needed to perform similarity transformations with respect to density, temperature and geometric size. It means that equations (3) and (4) will be solved for the gas that surrounds the clumps and consists mainly of light elements. Further, we will examine the clump evolution at a given evolutionary interval.

In order to perform spectral diagnostics of the supernova clumped debris for a given clump structure, it is convenient to consider the probability $\boldsymbol{P(z)dz}$ that a photon will survive in the clumped layer $\boldsymbol{z, z+dz}$ where $\boldsymbol{z}$ is the axis along the projected area of the ejecta with the number density of clumps $\boldsymbol{n_C}$, which is given by:

$$P(z)dz = e^{-\frac{z}{l}}\frac{dz}{l} \qquad (6)$$

where l is the mean free path of a photon through the ejecta,

$$l = \frac{1}{n_c \pi r_c^2} \quad (7)$$

According to Varosi & Dwek [6]

$$P(b)db = \int_0^{2R\sqrt{1-(\frac{b}{R})^2}} e^{-\frac{z}{l}} \frac{dz}{l} \quad (8)$$

Then, the fraction f_A of the projected area of the ejecta filled by clumps is given by the integral:

$$f_A = \frac{1}{\pi R^2} \int_0^R P(b) 2\pi b db \quad (9)$$

Thus, having known the fraction f_A and size distribution of the ejecta clumps, we can proceed to the reconstruction of the original picture of the supernova explosion. We consider that temperature and density obey to the similarity transformations; so does the linear size. Along with the time scale of the process, the cross section of individual clumps – $\boldsymbol{x}$ is also an independent dimensional variable involved in the similarity transformations. When solving this problem, we need to reduce the time scale and $\boldsymbol{r}$. The iteration process is to be terminated at the instant of time when the conditions of adiabatic compression are no longer met. This approach would be acceptable when the radiation does not influence the ejecta dynamics, i.e. when quanta are not confined in the matter. At the indicated instant of time, any of 100 clumps is noticeably affected by the shocks passed through the ejecta as these clumps are many orders of magnitude denser than the surrounding medium and remain intact being compressed by the blast waves produced in the supernova explosion. According to Varosi & Dwek [6], the fraction $f_A = 0.6 - 0.94$ with the total number of the ejecta clumps N_C=100 and N_C=1000 whereas the volume filling factor of the clumps in the ejecta $f_V = 0.1 - 0.8; 0.003 - 0.25$ with N_C=100 and N_C=1000, respectively. Further observations will enable us to improve the values f_A, f_V and N_C, although it appears that the defined problem can be solved using the above estimates, and the number of clumps can be determined even now.

4 Instability and clumps during the supernova explosion

One of the most meaningful conclusions drawn from the research of the supernova explosion is that hydrodynamic events when the iron core is being compressed by the infalling matter are of crucial importance (Popov *et al.*, [7]). It is a surprising fact that despite hundreds of different scenarios of the Type II supernova explosions suggested, the supernova energy eventually varies around the value of $E_0 \sim 10^{51}$ erg whereas the mass of the ejecta differ. The peculiarity of the SN 1987A explosion is the absence of the relativistic remnant that allows assuming the mass of the ejecta to be equal to $20\ M_\odot$. The Rayleigh-Taylor (RT) instability, arisen when the progenitor's envelope matter falls onto the dense core prior to the explosion, plays a crucial role in the elemental and spatial distribution of the debris. A detailed analysis of this RT instability based on the hydrodynamic and thermodynamic specific features of the physical processes at different times after the iron core collapse (Müller *et al.*, [9]) allowed determination of important relations for the kinematic properties of the initial clumps. First of all, we compare the criteria for such RT instability to grow for an incompressible gas,

$$\frac{\partial \ln \rho}{\partial r} / \frac{\partial \ln p}{\partial r} < 0 \quad (10)$$

as well as for a compressible gas under the same conditions and with the same adiabatic index:

$$\frac{\partial \ln \rho}{\partial r} / \frac{\partial \ln p}{\partial r} < \frac{1}{\gamma} \quad (11).$$

The growth rate of such an instability in the incompressible case is given by

$$\sigma = \sqrt{-\frac{p \frac{\partial \ln \rho}{\partial r} / \frac{\partial \ln p}{\partial r}}{\rho}} \quad (12)$$

For the compressible case it is given by

$$\sigma = \frac{c}{\gamma} \sqrt{\left(\frac{\partial \ln p}{\partial r}\right)^2 - \gamma \frac{\partial \ln p}{\partial r} \frac{\partial \ln \rho}{\partial r}} \quad (13)$$

where $\boldsymbol{c}$ is the adiabatic sound speed. Then, we introduce two amplitudes of the RT instability growing from the initial perturbation with amplitude ξ_0 at time t=0 to amplitude ξ at a given time $\boldsymbol{t}$.

In fact, the time scale of a RT instability active growth up to the final formation of an RT finger is 100 seconds. The main causes of the ejecta fragmentation into clumps are the presence of the RT instabilities and kinematic viscosity of the medium that result in the inhomogeneous density and elemental distribution of the ejecta. The values of the logarithmic derivatives in formulae (10)-(13)

are directly proportional to the dimensionless rates of the relevant thermodynamic values and sensitive to the non-linearity of the process.

Let us assume that all heavy elements comprising the core have been transferred to 100 fragments. From this distribution of elements the chemical composition of the clumps can be derived. Let us split the surface of the core into 100 fragments. Each fragment is formed as a clump due to the RT instabilities. Thermodynamic parameters inside such a clump formed due to the RT instability change in two stages. At the first stage, which is non-adiabatic, the clumps grow in size according to formulae (11) and (12) during 100 seconds. Then, on the time scales longer than 100 sec, the mean temperature $\boldsymbol{T(M_r, t)}$ and mean density of a clump $\boldsymbol{\rho(M_r, t)}$ can be calculated by formula (3) over a period till the current epoch. At the current epoch of observations, the clumps are clearly seen in the Spitzer infrared spectra. In the inter-clump medium, there are hydrogen, helium and other light elements comprising a substantial fraction of the ejecta. The observed structure of this population of fragments of debris formed in the explosion fits in a cylinder whose axis coincides with the pre-explosion rotational axis of the progenitor. Therefore, to perform mapping of the debris through its images, we are to detect its spectral manifestations according to the following scheme:

- all the fragments are distributed within a cylinder with the dimensions observed at the current epoch in the gamma-ray, X-ray, UV and IR bands;
- the fragment composition depends on the after-explosion elemental distribution (as shown in Figure 1, and Table 1 in Popov et al [7]);;
- individual fluorescence spectra of all fragments are defined by the non-thermal radiation field powered by the ionization loss of positrons emitted in a series of β-transformations $^{44}_{22}\mathrm{Ti} \xrightarrow{85\ \mathrm{y}} {}^{44}_{21}\mathrm{Sc} \xrightarrow{6\ \mathrm{h}} {}^{44}_{20}\mathrm{Ca}$, as well as by the chemical composition of each fragment [2].

To make further calculations, we need to compute the radiation field of several standard clumps with a given composition that contain a condensed phase consisting of a mix of astronomical silicates and graphite. Thus, in this section we have determined the spectroscopic manifestations of asymmetricity of the Type II SN 1987A explosion which resulted in inhomogeneous distribution of elements in the debris. The fragments in the polar and equatorial regions of the remnant are expected to exhibit the abundance of heavy elements and products of their nuclear fission, including $^{44}_{20}Ca$, which will be sufficient for their detection. Imaging of the ejecta in the form of clumps depending on their initial spatial and elemental distribution is illustrated in Figure 1.

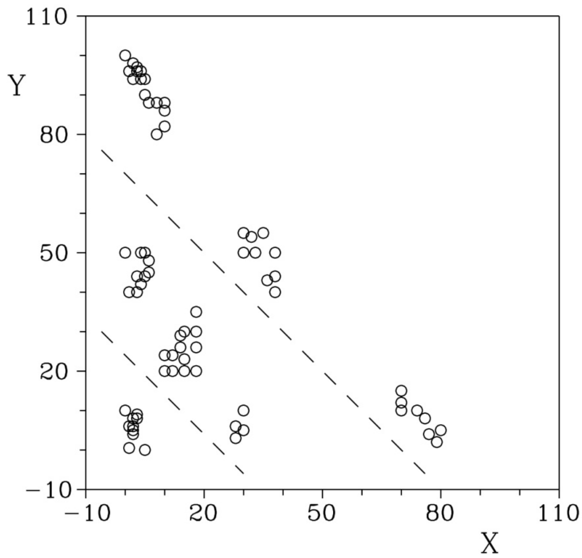

Figure 1 – The spatial model of the ejecta at three different epochs, namely near the zero time, after 15 years, and after 31 years. The groups are divided by dashed lines. The units of the axes are 10^{13} meters. The horizontal axe is the direction to equator, the vertical – to the pole. The points mark the places where $^{44}_{22}T\iota$ molecules are concentrated

5 Mapping of the images of the SN 1987A ejecta clumps

As is shown in the previous sections, both imaging of the remnant at the current epoch and its future mapping at different wavelengths are defined by its geometry, physical structure and chemical composition. As of today, high-quality images of two polar and one equatorial shock surfaces have been obtained. The data from the shock fronts at the nearest epoch will be used to study the circumstellar gas morphology, as well as the debris of the cocoon from which SN 1987A was formed. Modelling the remnant image at different time scales is mandatory for the determination of the spatial distribution of elements in the ejecta. Unlike the supernova remnants with a compact source, the expanding clumped ejecta of SN 1987A demonstrate their own emission throughout the volume. In these clumps, the main contribution to the radiation is made by two dust species (graphite and astronomical silicates) and atoms excited by fast positrons. As reported in Jerkstrad [1] and Doikov *et al*., [17-19], the spectral composition of the radiation emitted by a clump depends on its composition without regard to the dust. Modern observational data obtained by the Spitzer Space Telescope are sufficient to spatially resolve images of the ejecta clumps while the properties of these clumps are defined by their dust content. The predominance of astronomical silicates results in decreased absorption of visible and UV radiation. Graphite does not let either optical nor UV radiation through. However, γ and X quanta generated in the annihilation of fast positrons are poorly absorbed. In the progenitor, the abundance of oxygen outweighed that of hydrogen. Hence, as in Dwek & Arendt [10], Osterbrok [11], Lucy [12] the estimated composition of the dust grains favors the prevalence of astronomical silicates. The calculations of the Type II supernova explosions yielded the mean chemical composition of the remnants presented in section V. After the passage of positrons, each of the reduced atoms undergoes cascade transitions and emits the Auger electrons. Let us add dust grains with steady-state IR emission to these atoms; hence, we assume that $\boldsymbol{\tau_l}$ is the optical depth of $\boldsymbol{l}$ absorbers and emitters. Taking into account the results of the study of the optical spectra of SN 1987A, the recorded evidence of the presence of clumps, as well as physical conditions inside the clumps, indicates that the graphiteous dust emission is present at the distant shock fronts while astronomical silicates are concentrated inside the ejecta clumps. These silicates have been formed in the expanding ejecta under conditions described in the study by Kozasa [15], and their emission has been recorded by the Spitzer Space Telescope. The data on the radiation emitted by the dust show an excess in their IR emission due to additional heating by the Auger electrons and secondary short-wave quanta. Silicates add to the mean optical depth of the indicated $\boldsymbol{l}$ absorbers. Taking into account the absence of a central source of emission, we are to use the Osterbrock-Lucy formula [11], [12] for the emitters and absorbers uniformly distributed in the emitting volume:

$$\mathcal{P}_{esc}^{unif}(\tau,\omega) \equiv \frac{P_e(\tau)}{1-\omega[1-P_e(\tau)]} \quad (18)$$

$$P_e(\tau) = \frac{3}{4\pi}\left[1-\frac{1}{2\tau^3}+\left(\frac{1}{\tau}+\frac{1}{2\tau^2}\right)e^{-2\tau}\right] \quad (19)$$

where $\tau_{ext} = \tau_{abs} + \tau_{scat}$ and $\omega = {}^{\tau_{scat}}/_{\tau_{ext}}$.

Assuming that $\boldsymbol{\varepsilon}$ is the emission per unit volume of a clump and using formulae (18) and (19), Dwek & Arendt [10] have obtained formulae of the intensity $\boldsymbol{I_{out}(\tau,\theta)}$ and total emerging flux $\boldsymbol{F_{out}(\tau)}$, which will be convenient for further calculations:

$$I_{out}(\tau,\theta) = \frac{\varepsilon}{\rho\kappa}\left(1-e^{-2\tau\cos(\theta)}\right) \quad (20)$$

$$F_{out}(\tau) = \frac{\pi\varepsilon}{\rho\kappa}\left(1-\frac{1}{2\tau^3}+\left(\frac{1}{\tau}+\frac{1}{2\tau^2}\right)e^{-2\tau}\right) \quad (21)$$

For the ejecta clumps with different composition the total emerging flux $F_{out}(\tau)$ depends on the ratio between the scattering, absorption and emission coefficients. The structure of the radiation field of clumps containing no dust has been described in Doikov (2017), including the definition of the source function S (E,r). The presence of a substantial amount of the graphite dust in the debris should have noticeably suppressed the short-wave part of S (E,r) that could have made the formation of the remnant atomic spectra impossible; though, in fact, those spectra were observed in 1995. The graphite dust contributes to the emission at the shock fronts of SN 1987A whereas the silicate dust emission fits better to explain the spectroscopic features of the ejecta. The particle-size distribution function n(a) $\approx a^{-3,5}$; $10^{-9}m < a < 0.25\cdot 10^{-6}m$ is typical for the silicate dust.

When interpreting the spectroscopic data for the interstellar medium and supernova remnants, we consider the so-called astronomical silicates. Apart from their own emission, these particles scatter the short-wave component of S (E,r) and let the optical and IR emission through. When calculating monochromatic optical depth of a clump for a given S (E,r), we can add the formula for the emerging flux produced by astronomical silicates of a given type to formula (21) for the total emerging flux $F_{out}(\tau)$:

$$F_{\lambda}^{sil}(\lambda) = 4\pi m_d k(\lambda) B_{\lambda}(T_d) \qquad (22)$$

In this formula, $\boldsymbol{m_d}$ is the mass of silicate dust in the ejecta clump; $\boldsymbol{k(\lambda)}$ is the absorption coefficient for a given type of astronomical silicates; $\boldsymbol{B_{\lambda}(T_d)}$ is the Planck function for particles with temperature $\boldsymbol{T_d}$. Spectroscopic images of the ejecta clumps depending on S(E,r) by the values of the emerging fluxes $F_{out}(\tau)$ and $F_{\lambda}^{sil}(\lambda)$.

6 Formation of spectral lines in the clumpy structures of the supernova remnants

The specific physical feature of the ejecta clumps is their emission throughout the entire volume, i.e. the uniform distribution of the emitters and absorbers in the ejecta. At the same time, there is no reversing (a relatively cold) layer in the ejecta. Media with such a layer include, for instance, stellar chromospheres that have relatively low gas density. In this case, the transitions induced by the collisions of atoms with molecules in the ejecta are unlikely to occur. These collision-induced transitions, as mentioned in the previous sections, are caused only by positrons due to their considerable mobility. According to Jerkstand [1], the excitation of the optical transitions of atoms is only due to radiation. To make a spectral line prominent, the radiative excitation rate should be consistent with the optical cascade rates. This problem can be solved if S (E,r) has been preliminary defined. The rate of excitation quanta formation is defined by the number of collisions of positrons with atoms per unit of time z per unit volume z $=< v >/< \lambda >$, as well as by the composition and geometric characteristics of the clumps. In this case, we assume that the debris is shaped as a prolate spheroid. Detailed positional radio-astronomical observations of the remnant of SN 1987A, along with the data obtained by the Hubble and Spitzer Space Telescopes, INTEGRAL, etc., allowed of determination of the three-dimensional structure of the ejecta and shock surfaces. As a result, the cosines of the angle between the projected plane and normal to the line of sight have been determined: $i_x = 41^0, i_y = -8^0$, $i_z = -8^0$. The eccentricity of the spheroid is 1.18.

In the review by Potter *et al*. [4], the quantitative parameters of the remnant were calculated from the radio-astronomical data; that allowed obtaining the gradients of the main thermodynamic parameters defined over the width between the inner and outer regions of the ejecta:

$$\nabla \mathrm{n} = \frac{(10^8 - 10^5)(m^{-3})}{\Delta r\ (m)};$$

$$\nabla \mathrm{P} = \frac{(10^{-15} - 10^{-2})Pa}{\Delta r(m)}; \qquad (23),$$

$$\nabla \mathrm{T} = \frac{(10^{-2} - 10^{6})K}{\Delta r(m)}$$

where ∇n, ∇P and ∇T are the number density, pressure and temperature gradients, respectively, defined over the width between the inner and outer regions of the ejecta. For the entire remnant the Δr value is $\Delta r \approx 0.5 \cdot 10^{16} m$. If the observed fragmentation into clumps occurs, then value Δr makes $\Delta r \approx 0.5 \cdot 10^{16} m / N_c$. In this case, the given gradients in N_c clumps affect the dynamic evolution of the clumps in the proper reference frame. As time progresses, the clumps expand with consequent decrease in the number density, temperature and pressure. Factoring in the radioactive decays in a series of β-transformations $^{44}_{22}\mathrm{Ti} \xrightarrow{85\ \mathrm{y}} {}^{44}_{21}\mathrm{Sc} \xrightarrow{6\ \mathrm{h}} {}^{44}_{20}\mathrm{Ca}$ results in additional ionisation of the matter and in increased mean thermodynamic temperature Sigmund [3]. According to the INTEGRAL observations (Grebenev, [16]), the total mass of the radioactive isotope of titanium $^{44}_{22}Ti$ was about $10^{27} - 10^{28}$ kg during the period of observations. Then, the mass of $^{44}_{22}Ti$ confined in the clumps should account for their size distribution within the volume of the spheroid debris.

We have taken into consideration the results of numerous calculations using the explosion models of the progenitor of SN 1987A which prove that an essential fraction of the clumps, and hence heavy elements in these clumps – the nucleosynthesis products, is concentrated in two near-polar and narrow equatorial regions. The bulk of the progenitor's matter after its explosion is concentrated in the form of light elements in the inter-clump medium and outer regions of the debris. The clumps due to their significant inertness are concentrated in the inner part of the ejecta. To construct a model of the bolometric luminosity of the remnant, it is necessary to estimate the emission from each clump and then integrate it with respect to all geometric and physical specific features. All spectral line profiles also depend on the results of this mapping. In all clumps containing the major fraction of heavy elements the abundance of ${}^{44}_{20}Ca$ has increased by 30% over last 31 years, and it has been taken into account when calculating the optical characteristics of the clumps in the debris.

The integrated source functions and electron velocity distribution functions for the indicated main clumps in the ejecta are presented in Figs. 1-3 in Doikov et al [19] illustrate how the chemical composition of the clumps and the presence of dust influence the H and K ${}^{44}_{20}Ca$ II line profiles.

The presence of rather intense fluxes of positrons and electrons with the kinetic energies 10^3 times higher than the excitation energies for the H and K CaII, as well as neutral Ca I lines, leads to the formation of the lines of these elements. The second source of excitation is the hard radiation field from the iron-peak atoms formed in the cascade transitions after the passage of fast positrons. All these processes are important after the hydrogen recombination. A rather high degree of vacuum in the nebular remnant is offset by intense emission of the secondary Auger electrons and photons. The values of the resolved radiation function S (E,r) and electron velocity distribution function F (E,r) are given for the clumps with a defined spherical shape and composition. With the defined functions S (E,r) and F (E,r) the radiative energy balance for the H, K CaII and Ca I lines can be calculated. Given that the clumps were formed at one and the same instant of time, the model Ca line profile can be determined for each clump.

7 Conclusions

In the present paper, all the results have been obtained with an assumption that the young remnant of SN 1987A contains an important energy source, namely radioactive isotope of titanium ${}^{44}_{22}Ti$. Generating substantial fluxes of fast positrons, the remnant emits radiation throughout the entire volume of the ejecta clumps and inter-clump matter [5]. We have shown that all the clumps can be split into two groups. These groups are related primarily to the asymmetry of the supernova explosions. The majority of the ejecta wq clumps (among those 100 clumps which have been already detected) contain practically all iron-peak elements whereas the remaining clumps consist of C-N-O elements. Hydrogen and helium are concentrated in the frontal regions of the remnant, as well as in the interclump medium in the ejecta.

We suppose, according to formulae (14)-(17), that the equatorial part of the debris consists predominantly of the heavy iron-peak elements, including radioactive ones, whereas the clumps containing C-N-O elements are located in the polar regions of the ejecta. The silicate dust was formed mainly in the clumps in the equatorial region of the debris, and its fluorescence is caused by the local radiation field and fast positrons. Here we also expect the conditions for the formation of the ${}^{44}_{20}Ca$II emission lines and excitation of rotational transitions of the H_2 molecule in the inter-clump region.

The radiation field energy is sufficient for detection of the titanium molecules. However, their detection and identification requires additional observations and calculations which are to be the subject of our future papers. Intense formation of the graphite dust occurred in the polar regions of the ejecta. Its presence in the clumps and around them results in blocking out the short-wave UV and optical radiation. The fact that SN 1987A produces optical spectra indicates that the graphite dust is concentrated in certain regions of the ejecta or that it can be observed due to its heating at the shock fronts. Figure 1 illustrates the evolution of the emerging flux due to the growth of the ejecta clumps.

Radio-, IR and X-ray observations of the remnant of SN1987A have shown a decrease in the IR and UV fluxes and an increase in the radio flux. This evidence suggests that the fluxes emerging at the shock fronts have been decreasing, hence one can learn about the remnant only by examining its own energy sources.

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