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Description of the related and resonant states of the nuclei $^{6}_{3}$Li of the lie in the method of resonant groups

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To consider the interactions between light nuclei, as well as the nature of the nuclear forces between them, and the test was made of the coupled and resonant positions of the nucleus $^{6}_{3}$Li using the technique of the "Algebraic version of the resonant group method (AV RGM)", which takes into account the Pauli principle, and the cluster approximation. As a result, the nucleus $^{6}_{3}$Li was considered as a bound and resonant position formed by the interaction of two clusters. This approach can greatly facilitate the ongoing calculations, reducing the interaction between the set of nucleons, to the interaction of individual clusters. That's way; the calculations carried out are an example of the applicability of this technique for studying interactions between light nuclei in the low-energy region up to 10 MeV. Also, special attention has the influence of nuclear forces, which are used for the description different nucleon-nucleon potentials. Consideration of the interaction between two clusters using alternately several different nucleon-nucleon potentials will allow us to better understand and determine the nature of nuclear force, which will be manifested through the obtained parameters of the bound and resonance positions, as well as the type of the built-up phase shifts. All calculations were performed using different values of nuclear parameters and quantum numbers.

Key words: light nuclei, clusters, bound and resonance states.
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1 Introduction

The isotope of the nucleus $^{6}_{3}$Li refers to even-odd light nuclei, which are often used as a visual and effective representation of these nuclei as cluster systems [1, 2, 3, 4] for the purpose of testing and conducting methods for calculating the properties of these nuclei by modern computational programs.

Corresponding calculations were carried out taking into account cluster representations and microscopic methods [5] of the group of nuclei. Earlier, the characteristics of the nuclei were researched: $_{2}$He^{5}, $_{3}$Li^{5} and $^{6}_{3}$Li [6]. The results obtained were in agreement with earlier experimental data [7]. Moreover, a description was given of the processes associated with interactions of clusters inside the nucleus, and the interaction of nucleons with these clusters, as well as the interaction of incident particles on the cluster core [8].

When considering the states and structure of the nucleus$^{6}_{3}$Li, the interaction forces between cluster components and nucleons inside a given nucleus are of particular importance, taking into account the values of their quantum numbers: rear s, orbital angular momentum L, parity π and angular momentum J. Taking into account the peculiarities of the interactions between nucleons and clusters, three kinds of nucleon-cluster potentials were used in the proposed work: modified Hasegawa-Nagata [9, 10], potential Volkov's V2 [11] and potential of Minnesota [12]. Each of these potentials has its own peculiarities; in particular, the first of them takes into account the model three-component interaction of an alpha particle with a deuteron, including Coulomb, centrifugal and spin-orbital forces between these clusters, Volkov's potential takes into account the two-component interaction of clusters, and the Minnesota potential is based on three potentials of the

Gaussian type. Into these potentials are included the values of exchange waves for a more accurate description of nuclear interactions between nucleons and clusters.

Note that the "algebraic version of the method of resonant groups" (AV RGM) [13, 14, 15], first proposed by G. F. Filippov, proved like very effective and has been widely used in studies of the properties of various cluster structures.

This method was used by us in this work, and became the basis for the calculation programs used by us. Such a calculation technique, called "2cl_SpectrPhases.exe", was further improved by Vasilevskiy V.S. – one of the co-authors of this work [13, 14]. In this way, AV RGM is now one of the widely used methods for calculating microscopic states in cluster models. In particular, the nucleus ${}^{6}_{3}\mathrm{Li}$ was considered as the state of the interacting clusters α and d:

$$\alpha + \mathrm{d} \rightarrow {}^{6}_{3}\mathrm{Li}^{*} \rightarrow \alpha + \mathrm{d} \qquad (1)$$

Such a representation allows us to consider instead of a large number of nucleons involved in the reaction only two compact clusters. In this way, problems with a large number of interacting nucleons can be reduced to the problem of two or three bodies, which in many respects simplifies the scheme of calculating the computing time of computer systems without losing sufficient accuracy of calculations.

This method was used for numerical calculations of the reactions using the calculation program proposed here, which makes it possible to investigate in detail the amplitudes and phases of the scattering of reactions, such as reactions (1). The obtained data showed good consent with the experimental values [7]. All calculations were carried out in the low-energy range from 0 to 10 MeV, in the program "2cl_SpectrPhases.exe" programmed to calculate the reactions involving light nuclei.

2 Principal

The considered elastic scattering reactions (1) with the participation of two clusters and the formation of the nucleus ${}^{6}_{3}\mathrm{Li}$ were performed in the program "2cl_SpectrPhases.exe" for given initial parameters, indicating the type of interacting clusters (Table 1).

Table 1 – Types of clusters considered by the program "2cl_SpectrPhases.exe" for two cluster reactions

Clu_name_1	${}^{4}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$
Clu_name_2	n	p	2n	d	t	${}^{3}_{2}\mathrm{He}$	${}^{4}_{2}\mathrm{He}$

Assuming reactions, the program always takes the first particle cluster α, using as an incident cluster the particles indicated in the tables under the group: Clu_name_2, in particular, in the framework of this work, an incident particle chooses to be deuterium d.

Then, quantum numbers of interacting clusters are inputs, which are install parameters (Table 2).

Table 2 – Nucleus and basic input parameters

Nucleus	${}^{5}_{2}\mathrm{He}$	${}^{5}_{3}\mathrm{Li}$	${}^{6}_{2}\mathrm{He}$	${}^{6}_{3}\mathrm{Li}$	${}^{7}_{3}\mathrm{Li}$	${}^{7}_{4}\mathrm{Be}$	${}^{8}_{4}\mathrm{Be}$
Na	5	5	6	6	7	7	8
tot_spin	0.5	0.5	0	1	0.5	0.5	0

In addition to the install parameters showed in Table 2, in configuration file "2cl_calc_spec.cfg", sets the parameters:

- lm – orbital angular momentum
- tot_mom – total angular momentum of the system;
- r_0 – oscillator length;
- majoran – Majorana parameter, which sets by a definite value for each of the potentials in this work. In this case, the modified Hasegawa-Nagata potential equal to – 0.0009. The Majorana parameter

chooses to reproduce the experimental value of the ground-state energy of the nucleus;

- n_ob_funs – parameter that established the number of basic functions which the wave function of the relative motion of clusters. Calculations were conducted by 200 basic functions;
- npot – parameter responsible for the selection of the nucleon-nucleon potential, which will be used in the calculations (Table 3). Their classification is embedded in the program, where each of the potentials was marked with its number. At the work used: modified Hasegawa-Nagata potential (npot = 1), Volkov's potential B2 (npot = 3) and the potential of Minnesota (npot = 6);

Table 3 – Classification of nucleon-nucleon potentials [4]

npot	Name of potentials
1	Modified Hasegawa-Nagata potential [10]
2	Volkov's potential V1 [11]
3	Volkov's potential V2 [11]
4	Brink-Boker potential B1
5	Brink-Boker potential B2
6	potential of Minnesota [12]

- E_ini, E_fin – parameters that determine the energy interval in which the phases and scattering cross sections will be calculated. The parameter E_ini specifies initial energy, E_fin sets the final energy. In this work the scattering reaction is α + d will be considered in the energy range from 0 to 10 MeV. In this connection, the parameters will have values: for start energy E_ini = 0, for final energy E_fin = 10 MeV;

After entering all the install parameters for reactions (1). Program start to calculation which task is: calculation of the Hamiltonian of two cluster systems$\hat{H}$, and phase-scattering related, also resonant positions. As a result, the wave function describing the positions of two clusters and their interactions with each other will be represented in the form [13, 14]:

$$\Psi_J = \hat{A}\{[\varphi_1(A_1)\varphi_2(A_2)]_s\psi_{LS}^J(\vec{q})\}, \qquad (2)$$

where $\hat{A}$ this is an operator of the antisymmetrization – indistinguishability system from A nucleons by permutations of all nucleon pairs

$$\hat{A} = \sum_{p=1}^{N!}\varepsilon_p\hat{P_p}, \qquad (3)$$

$\hat{P_p}$ – an operator that performs a permutation on A nucleons, $N!$ – the sum of all permutations in A of the particles, $\varepsilon_p = \pm 1$ – the sign of this permutation, where +1 – for an even permutation, -1 – for an odd permutation.

$\varphi_1(A_1)$ – internal wave function of nucleons(A_1) in first α – cluster, dependent on the coordinates of the first four of nucleons – $\overrightarrow{r_1}$, $\overrightarrow{r_2}$, $\overrightarrow{r_3}$, $\overrightarrow{r_4}$.

$\varphi_2(A_2)$ – internal wave function of nucleons (A_2) in the second d-cluster, which depends on the coordinates of the second part of the nucleons – $\overrightarrow{r_5}$,$\overrightarrow{r_6}$.

$\psi_{LS}^J(\vec{q})$ – the wave function of the relative motion of two clusters, in this work the alpha deuterium, which depends on the Jacobi vector.

$\vec{q}$ – the Jacobi vector, which is proportional to the vector $\vec{r}$ [8, 13, 14, 15]

$$\vec{q} = \vec{r}\sqrt{\frac{A_1A_2}{A_1+A_2}}, \qquad (4)$$

where $\vec{r}$ – determines the relative distance between the centers of mass of interacting clusters [8, 13, 14].

$$\vec{r} = \left[\frac{1}{A_1}\sum_{i\in A_1}\vec{r}_i - \frac{1}{A_2}\sum_{J\in A_2}\vec{r}_j\right], \qquad (5)$$

where $\vec{r}_i$ – it's coordinate of i nucleons (i = 1, 2, 3, …, A_1) from first cluster, and $\vec{r}_j$ – it's coordinate j nucleons ($j = A_1$+1, A_1+2,…, $A_1 + A_2$) from second cluster.

Internal wave functions $\varphi_1(A_1)$ and $\varphi_2(A_2)$, describing the motion of nucleons within both clusters, within the framework of the cluster approximation and the microscopic method of AV RGM, must be fixed, and constructed in the form of Slater determinants from the oscillator functions of the translationally invariant shell model [15, 16]. As a result, the functions $\varphi_1(A_1)$ and $\varphi_2(A_2)$ depend from oscillator length of r_0, inputted with install parameters.

Carrying out calculations using AV RGM, the wave function of the relative motion of the clusters $\psi_{LS}^J(q)$ can be reduced to a simple algebraic form, using for this purpose the expansion of the intercluster function with respect to the complete set of $\psi_n(q, r_0)$ of normalized radial oscillator functions in the coordinate and momentum spaces [13].

$$\psi_{LS}^J(q) = \sum_{n=4}^{\infty} C_{nL}\,\psi_{nL}(q, r_0), \qquad (6)$$

$$\psi_{LS}^J(\mathrm{p}) = \sum_{n=4}^{\infty} C_{nL}\,\psi_{nL}(\mathrm{p}, r_0). \qquad (7)$$

where q – vector modul $\vec{q}$ [13],

$$\psi_{nL}(q, r_0) = (-1)^n N_n r_0^{-\frac{3}{2}} \rho^L e^{-\frac{\rho^2}{2}} L_n^{L+1/2}(\rho)^2, \quad (8)$$

$$\rho = \frac{q}{r_0}, \qquad N_n = \sqrt{\frac{2\Gamma(n+1)}{\Gamma(n+L+\frac{3}{2})}},$$

$$\psi_{nL}(\mathrm{p}, r_0) = N_n r_0^{\frac{3}{2}} \rho^L e^{-\frac{\rho^2}{2}} L_n^{L+1/2}(\rho)^2, \qquad (8.5)$$

$$\rho = \mathrm{p} \cdot r_0,$$

where n – number of oscillator quanta (knots),r_0 – oscillator radius, G – known gamma function [17], $L_n^{L+1/2}$ – generalized Laguerre polynomial [18], C_{nL} – Fourier coefficients.

After that, the recording of formula (2) of the total wave function of two cluster systems in the algebraic version of the method of resonant groups takes the form of a generalized Fourier series [13, 16, 19]

$$\Psi_J = \sum_{n=4}^{\infty} \mathrm{C}_{nL}\, \Psi_{nL}, \qquad (9)$$

where

$$\Psi_{nL} = \hat{A}\{[\varphi_1(A_1)\varphi_2(A_2)]_s \psi_{nL}(q, r_0)\}, \qquad (10)$$

Ψ_{nL} – This is the basis of many-particle oscillator functions, which is used to describe this cluster system of light nuclei.

Due to the presence of correct boundary conditions, this method is well used for describing the position of both continuous and discrete spectra. A feature of the AV RGM is the reduction of the problem of finding the wave function of the relative motion of clusters $\psi_{LS}^J(\vec{q})$ to the problem of finding the unknown coefficients of the Fourier expansion C_{nL}[11]. What is the wave function of the relative motion of two clusters in a discrete oscillator representation, satisfying a system of linear algebraic equations [19, 20]

$$\sum_{m=4}^{\infty}[\langle \Psi_{nL}|\hat{\mathrm{H}}|\Psi_{mL}\rangle - \mathrm{E} \cdot \langle \Psi_{nL}|\Psi_{mL}\rangle]\mathrm{C}_{mL} = 0, \quad (11)$$

where $\langle \Psi_{nL}|\hat{\mathrm{H}}|\Psi_{mL}\rangle$ are the matrix elements of the Hamiltonian between the cluster oscillator functions. Dirac brackets mean integration over spatial coordinates and summation over spin and isospin variables of all nucleons. $\langle \Psi_{nL}|\Psi_{mL}\rangle$ – the normalization of nucleus or the overlap integral of the oscillator functions (10) [19, 20]

$$\langle \Psi_{nL}|\Psi_{mL}\rangle = \delta_{n,m}\lambda_n, \qquad (12)$$

where λ_n – eigenvalues of the antisymmetrization operator. At λ_n = 0 position Ψ_n is a prohibited position of Pauli. Such positions do not participate in the construction of the wave function (2) and do not describe the dynamics of two cluster systems. For this, positions only allowed Pauli principle are used, for which $\lambda_n > 0$.

Then, in carrying out the renormalization of the basis functions and the Fourier coefficients due to the influence of the antisymmetrization operator $\hat{A}$, the equation (11) was reduced to the standard matrix form of the Schrödinger equation with an orthonormal basis of the function. Because of this, has obtained endless system of linear homogeneous algebraic equations of the form [13, 20, 21]

$$\sum_{m=4}^{\infty}[\langle \bar{n}L|\hat{\mathrm{H}}|\bar{m}L\rangle - \mathrm{E} \cdot \delta_{n,m}]\bar{\mathrm{C}}_{mL} = 0, \quad (13)$$

where $\hat{\mathrm{H}}$ – multi particle nuclear Hamiltonian, E – total energy of the nuclear system, $\langle \bar{n}\mathrm{L}|\hat{\mathrm{H}}|\bar{m}L\rangle$ – matrix elements of the Hamiltonian on functions Ψ_{nL}, n and m – indexes that numbers only positions accepted by Pauli principle.

By this, the problem of solving the wave function of interacting clusters reduces to the matrix problem of the expansion of the inter cluster wave function over the complete system of oscillator functions, where the eigenvalues and eigenfunctions of the Hamiltonian. Negative eigenvalues determine the energy of bound positionsE_α, and the corresponding eigenfunctions determine the wave functions of the bound positions of Ψ_α nucleus.

The positive eigenvalues and eigenfunctions represent the state of the continuous spectrum of the nucleus.

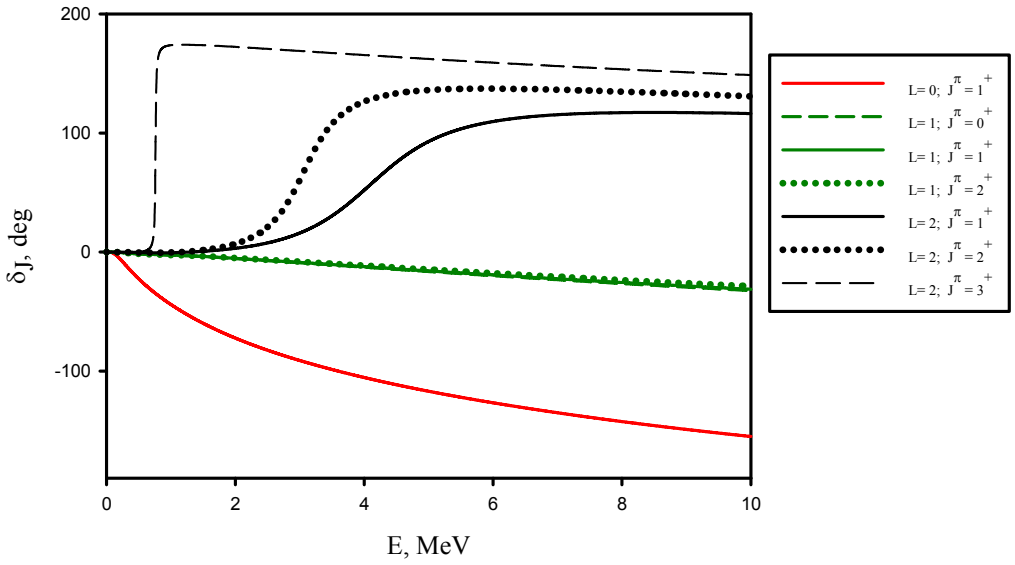

Figure 1 – The phase shifts for elastic scattering d + α constructed using the modified Hasegawa-Nagata potential

The phase shifts shown in Figure 1 were constructed using the modified Hasegawa-Nagata potential.

In the graph above (Figure 1), the built-up phase shifts showed the presence of bound positions for the values L = 0 and L = 1 through channels with values $J^{\pi} = 0^{+}$, 1^{+} , 2^{+} that they were marked with red and green lines. As the orbital angular momentum increases till L = 2 for channels with value of $J^{\pi} = 0^{+}$, 1^{+} , 2^{+} there appeared resonance positions marked with black lines. The presence of these resonance positions indicated an increase in the influence of the centrifugal barrier in the inter particle interaction and its role in the formation of resonance positions. Also, using the modified Hasegawa-Nagata potential, which takes into account not only the centrally nuclear but also the spin-orbit component, it is possible to analyze the effects of the spin component on the resonances formed and their lifetime. For what was taken into account the ratio of resonance energy to its width and lifetime: $E=\frac{\Gamma}{2}=\frac{\hbar}{2\tau}$, $\tau=\frac{\hbar}{\Gamma}$; where E – resonance energy, G – width of resonance, τ – lifetime of resonance.

The existence times of these resonance positions were determined. As a result, resonances for the values analyzed for $J^{\pi} = 1^{+}$, 2^{+} , 3^{+} showed an increase in the lifetime of the resonance with respect to the increasing value of its spin-orbit component. So it was calculated that the lifetime τ for the resonance with the value of $J^{\pi} = 3^{+}$ form $\tau = 3.43562 \times 10^{-21}$s. which was two orders of magnitude higher than the lifetime for the resonance with the value of $J^{\pi} = 1^{+}$ constituting $\tau = 2.792579 \times 10^{-23}$s. and $\tau = 6.49688 \times 10^{-23}$s. for $J^{\pi} = 2^{+}$ respectively. These values, like the phase shifts in Figure 1, showed the dependence of the resonance states and their parameters on the available quantum numbers.

Figure 2 shows the obtained phase shifts using the Volkov's potential B2.

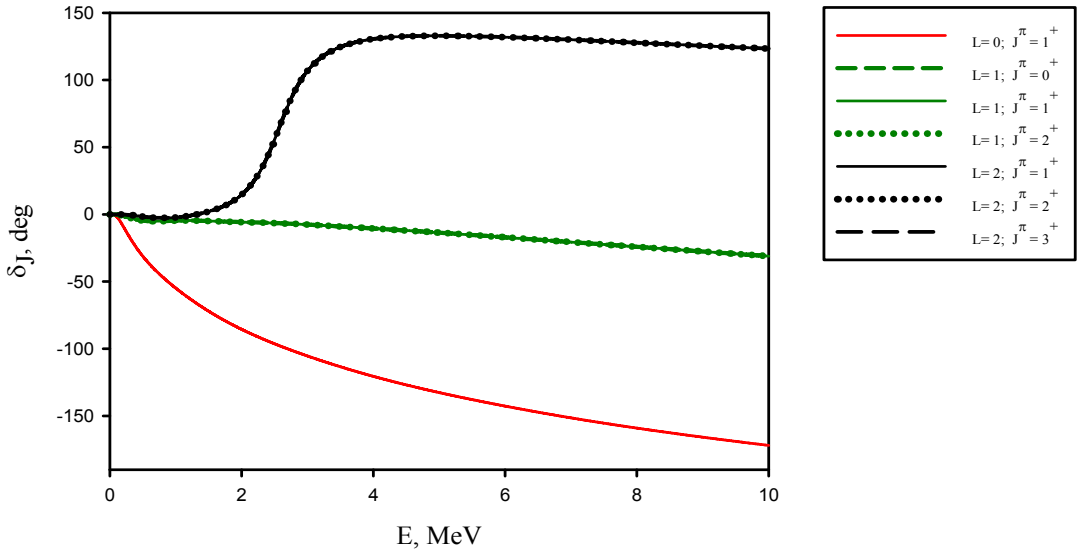

Figure 2 – The phase shifts for elastic scattering d + α constructed using the Volkov's potential V2

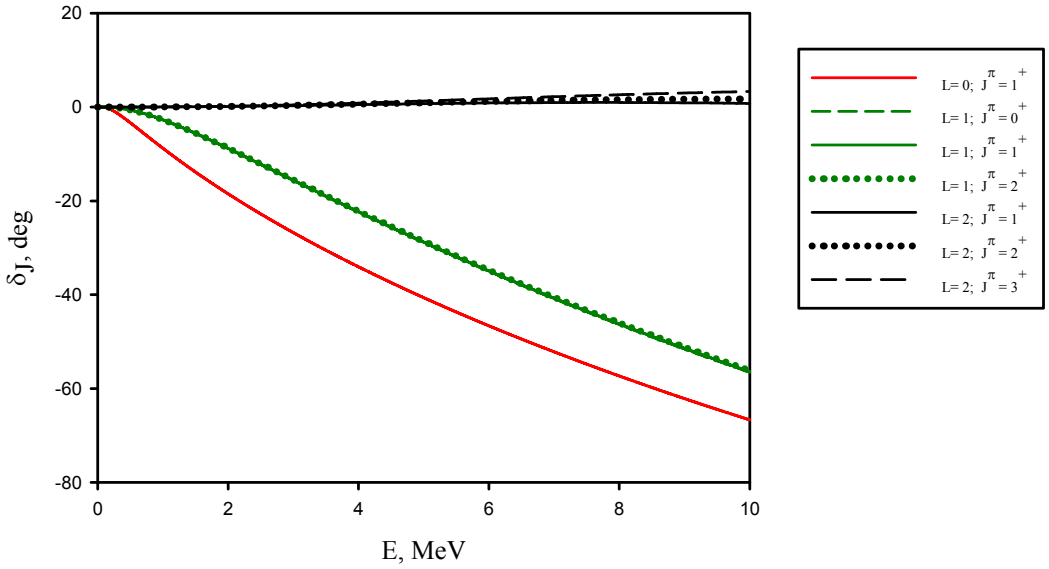

Figure 3 – The phase shifts for elastic scattering d + α constructed using the Minnesota potential

In Figure 3, the phase shifts were obtained using the Minnesota potential.

3 Results

In the result of calculations for the scattering of light nuclei α + d with the formation of bound and resonance positions in the form of a compound nucleus ^{6}Li, data were obtained describing these positions and the conditions for their appearance. All calculations were made at low energies from 0 to 10 MeV, using nucleon-nucleon potentials: Hasegawa-Nagata (modified) [10], Volkov's V2 [11] and Minnesota [12], each of which had its own exchange parameters and characteristics, which were clearly expressed in the results obtained. Due to this, the phases of scattering of bound and resonance positions were constructed, presented in Figures 1 – 3. The

scattering phase data calculated for elastic scattering of α + d with different values: of total orbital angular momentum L, total angular momentum J and definition π, which showed significant differences in the formed positions. What indicated the influence: Coulomb and centrifugal barriers on the lifetime of three formed resonance positions: $J^{\pi}= 3^{+}$, $J^{\pi}= 2^{+}$ and $J^{\pi}= 1^{+}$ at L = 2. Received scattering phases as well as the scattering phases constructed on the using of modified Hasegawa-Nagata potential showed the presence of resonant positions in the range of L = 2. However, the scattering phases obtained by using the Volkov's potential V2 shown in Figure 2 featured significant differences from our previous results obtained by using the Hasegawa-Nagata potential. It is important that in the case of the Volkov's potential there are no phase shifts in the resonances that were manifested in the case of the Hasegawa-Nagata potential. But in the case of the Volkov's potential, it turns out that the resonance generation energy for the positions with Jπ = 1+, 2+, 3+ is the same and equal to $E_{res.}$ = 2.567 MeV.

Such differences can be a consequence of the peculiarities of Volkov's' potential V2, which consists of two Gaussian functions and does not describe the interactions associated with the change in the spin and isospin parameters. In this regard, the graph presented in Figure 2 does not show changes in the scattering phases, which could be related to the influence of the spin component on them.

The Minnesota potential (Figure 3), like the modified Hasegawa-Nagata potential, takes into account the spin-orbit interaction, which is determined by three Gaussian functions. However, using the Minnesota potential to solve the problems posed and for construct the scattering phases obtained with it, the results doesn't showed the presence of resonant positions, but in this case they yielded results only for bound positions. The associated positions described by the Minnesota potential gave good consent with the results obtained using the modified Hasegawa-Nagata potential.

All values of the parameters of bound and resonant positions for different values of L and J^{π} were obtained by using various nucleon-nucleon potentials that were compared and were in consent with the experimental data. Results showed at Table 4.

Table 4 – Experimental and theoretical values of the parameters of bound and resonant positions using different nucleon-nucleon potentials

Potential	L; J^{π}	$E_{rel.}$(MeV)	$E_{res.}$(MeV)	G(MeV)
Modified Hasegawa-Nagata potential	0; 1^{+}	-1.432517	-	-
	1; 0^{+}	0.443289	-	-
	1; 1^{+}	0.443270	-	-
	1; 2^{+}	0.443229	-	-
	2; 1^{+}	0.607098	4.1	2.356595
	2; 2^{+}	0.606985	3.063	1.013097
	2; 3^{+}	0.601944	0.763	0.019158
Volkov's's potential V2	0; 1^{+}	-0.886018	-	-
	1; 0^{+}	0.403692	-	-
	1; 1^{+}	0.403692	-	-
	1; 2^{+}	0.403692	-	-
	2; 1^{+}	0.543601	2.567	0.900432
	2; 2^{+}	0.543601	2.567	0.900432
	2; 3^{+}	0.543601	2.567	0.900432
Minnesota potential	0; 1^{+}	0.348620	-	-
	1; 0^{+}	0.464726	-	-
	1; 1^{+}	0.464722	-	-
	1; 2^{+}	0.464715	-	-
	2; 1^{+}	0.640570	-	-
	2; 2^{+}	0.640567	-	-
	2; 3^{+}	0.640564	-	-
Experimental values [7]	0; 1^{+}	-1.4743	-	-
	2; 2^{+}	-	2.838 ±22	1.30 ± 100
	2; 3^{+}	-	0.712 ±2	0.024 ± 2

4 Conclusion

Calculations of the investigation of bound and resonance positions of the ^{6}Li nucleus by using the program "2cl_SpectrPhases.exe" showed the dependence of the resonant positions on the centrifugal and spin-orbit components. This dependence is particularly important in phase shifts and lifetimes of resonances with different quantum numbers. The presence of exchange parameters in the calculations and using of three nucleon-nucleon potentials made it possible to obtain the characteristics of connected and resonant positions.

It was obtained the magnitude of the exchange forces on the parameters of each of the potentials, calculated shifts of the scattering phases and an analysis of their dependences on the magnitude of the exchange forces. Thereby, was determined the potential of each components in the description of the formation of bound and resonant positions in the low-energy range.

Dispersion phases, formed as a result of work and the main characteristics of resonance states, obtained from calculations and formed in the field of low energy from 0 to 10 MeV, have given an essential information about the nature and influence of nuclear forces in system of interest. Also, comparing obtained theoretical results and experimental data, have shown applicability of the method used in this work and cluster approximation for description of reactions involving lightweight nucleus in the field of low energy.

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