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On three-cluster disintegration of ^{9}Be

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We study a process of the photodisintegration of some Borromean light nuclei into three clusters. This study is performed within a three-cluster microscopic model. In Reference [1] this model was applied to obtain parameters of resonance states in ^{9}Be and ^{9}B and to establish their nature. Main aim of the present investigations is to describe the dipole transition probability from the ground state of ^{9}Be to the states of three-cluster α+α+n continuum by using the same model. That model exploits the hyperspherical harmonics basis (HHB) and thus reduces many-channel Schrödinger equation to the algebraic matrix (AM) form. The dipole transitions from the ground 3/2- state to the 1/2+ states of three-cluster continuum were studied in detail. The role of resonance states in three-cluster continuum to this process is investigated in detail. The dominant channels with the maximal dipole strength due to the coupling between the ground and scattering states are discovered.

Key words: light nuclei, three-cluster microscopic model, hyperspherical harmonics basis, algebraic matrix, dipole transition.
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1 Introduction

We consider the photodisintegration of some light nuclei into three fragments. This process is considered for nuclei with distinguished three-cluster features, or, in other words, nuclei which have the lowest three-cluster decay threshold. Such nuclei are also called the Borromean nuclei. In the present paper we will concentrate on the nucleus ^{9}Be, which considered as a three-cluster configuration α+α+n. These investigations are also aimed at clarifying the existence and properties of the $1/2^{+}$ resonance state in ^{9}Be.

All investigations are carried out within a microscopic three-cluster model. For this model we will use an abbreviation AMHHB which indicates that the model exploits the hyperspherical harmonics basis (HHB) and thus reduces many-channel Schrödinger equation to the algebraic matrix (AM) form. The key elements of the AMHHB were formulated in Reference [2] and then applied to study the three-cluster continuum in light nuclei.

In a set of publications [3, 4, 5, 6, 7, 8] the photodisintegration of^{9}Be into two alpha-particles and a neutron has been considered within the orthogonality condition model (OCM) which make uses the Gaussian basis and the complex scaling method (CSM) to locate resonance states. In these papers the $1/2^{+}$ excited state was shown to be a virtual state situated a very close to the two-cluster ^{8}Be+n threshold. It was also shown that a huge peak of the photodisintegration cross section is created by the virtual state in the two-body channel ^{8}Be (0^{+})+n.

The three-cluster photodisintegration also attracts numerous experimental studies. The recent experimental measurements of the photodisintegration cross section are presented in References [9, 10, 11].

2 AMHHB and coupled channels methodology

In this section we present the main ideas of the AMHHB method. Basic ideas of the method were formulated in Reference [2]. In the present paper we will the same notations as in recent publications [1, 12, 13].

Any microscopic model is based on a microscopic Hamiltonian, which includes a nucleon-nucleon interaction and the Coulomb forces, and on form of a fully-antisymmetrized wave function.

Within a three-cluster model, a wave function of compound system with a partition A = $A_1 + A_2 + A_3$ is

$$\Psi_{EJM_J} = \hat{A}\Big\{\big[\Phi_1(A_1,s_1)\Phi_2(A_2,s_2)\Phi_3(A_3,s_3)\big]_S \quad (1)$$
$$\times \sum_{l_1,l_2} f^{(E,J)}_{l_1,l_2;L}(x,y)\left\{Y_{l_1}(\hat{\mathbf{x}})Y_{l_2}(\hat{\mathbf{y}})\right\}_L\Big\}_{JM_J}$$

All notations are the same as in Reference [1]. We also refer to Reference [1] for explanation of details of all parts of the wave function (1), quantum numbers and the Jacobi vectors **x** and **y**.

By using hyperspherical coordinates

$$x = \rho\cos\theta,$$
$$y = \rho\sin\theta, \quad (2)$$
$$\Omega = \{\theta, \hat{\mathbf{x}}, \hat{\mathbf{y}}\},$$

the wave function (1) is represented as

$$\Psi_{EJM_J} = \hat{A}\Big\{\big[\Phi_1(A_1,s_1)\Phi_2(A_2,s_2)\Phi_3(A_3,s_3)\big]_S \times \quad (3)$$
$$\times \sum_{K,l_1,l_2,L} \psi^{(E,J)}_{K,l_1,l_2;L}(\rho)\, Y_{K,l_1,l_2;L}(\Omega)\Big\}_{JM_J},$$

where K is the hypermomentum, $Y_{K,l_1,l_2;L}(\Omega)$ is a hyperspherical harmonic. A set of quantum numbers

$$c = \{K, l_1, l_2, L\}, \quad (4)$$

numerates channels of the three-cluster continuum.

The hyperspherical harmonics allow us to employ the rigorous methodology coupled channels. In this case the many-particle Schrödinger equation transforms in a set of coupled equations for a column vector of hyperradial functions:

$$\mathrm{x}\,\psi = \begin{pmatrix} \psi_{c_1} \\ \psi_{c_2} \\ \psi_{c_3} \\ \vdots \end{pmatrix}, \quad (5)$$

Components $\psi_{c_i}(\rho)$ of the wave function (5) are subject for boundary conditions, which were discussed in detail in Reference [12]. The total many-particle Hamiltonian $\hat{H}$ is split onto the channel Hamiltonians $\hat{H}_{cc}$ and $\hat{H}_{c\tilde{c}}$ ($\tilde{c} \neq c$) for coupling between channels. Both channel Hamiltonians and coupling Hamiltonians contain local and non-local componentsdue to the antisymmetrization. In the channel Hamiltonians, the local part consists of the kinetic energy operator and the folding (or direct) potentials. On other hand, the coupling Hamiltonians contains the local part consisting of the folding (or direct) potentials only. It means that there are contributions from the interaction only, but not from the kinetic energy term.

Hamiltonians $\hat{H}_{cc}$ and $\hat{H}_{c\tilde{c}}$ ($\tilde{c} \neq c$) are obtained by sandwiching of the total Hamiltonian between the corresponding hyperspherical harmonics Y_c and $Y_{\tilde{c}}$

$$\left\langle \hat{A}\{\Phi_1\Phi_2\Phi_3 Y_c\}\middle|\hat{H}\middle|\hat{A}\{\Phi_1\Phi_2\Phi_3 Y_{\tilde{c}}\}\right\rangle$$

and integrating over all hyperangels and over those Jacobi coordinates describing the internal structure of interacting clusters. If we assume that the antisymmetrization operator $\hat{A}$ = 1, we obtain the local form of the Hamiltonians $\hat{H}_{c\tilde{c}}$.

It is important to underline, that within the coupled channel methodology, if we have N_{ch} open channels then for each energy we have N_{ch} independent solutions (wave functions) which describe all possible elastic and inelastic processes. It is well known (see for instance, chapter 6 of book [14] and Reference [15]) that the first wave function is obtained by assuming that there is an incoming wave in the first channel and outgoing waves appear in all channels, the second wave function contains the incoming wave in the second channel, and so on. Thus these functions can be marked by the channel c which possesses both incoming and outgoing waves. Thus, considering a photo- or electro-disintegration of Borromean nuclei we automatically obtain N_{ch} cross sections of the process.

Suppose we obtained wave function of a bound state $\Psi_{E_iJ_i}$ and wave function $\Psi_{E_fJ_f}$ of continuous spectrum state with energy E_f measured from the three-cluster threshold. Then we can calculate probability of the dipole λ=1 transition from bound to continuous states

$$B\left(E\lambda;E_i,J_i \Rightarrow E_f,J_f\right)= \\ =\frac{1}{2J_i+1}\left|\left\langle E_f,J_f\left\|\hat{Q}_\lambda\right\|E_i,J_i\right\rangle\right|^2, \qquad (6)$$

where

$$\hat{Q}_{\lambda\mu}=\sum_{i=1}^{A}\frac{1}{2}\left(1+\hat{\tau}_{iz}\right)r_i^\lambda Y_{\lambda\mu}\left(\hat{\mathbf{r}}_i\right), \qquad (7)$$

vector $\mathbf{r}_i$ ($\mathbf{r}_i=r_i\hat{\mathbf{r}}_i$) is a coordinate of the i^{th} nucleon. It is important to note that wave functions $\Psi_{E_iJ_i}$ and $\Psi_{E_fJ_f}$ of bound and scattering states, respectively, are normalized by the following conditions

$$\left\langle\Psi_{E_i,J_i}\middle|\Psi_{E_i,J_i}\right\rangle=1, \qquad (8)$$

$$\left\langle\Psi_{E_f,J_f}\middle|\Psi_{\tilde{E}_f,J_f}\right\rangle=\delta\left(k-\tilde{k}\right), \qquad (9)$$

where

$$k=\sqrt{\frac{2mE}{\hbar^2}}, \quad \tilde{k}=\sqrt{\frac{2m\tilde{E}}{\hbar^2}}.$$

In this section we defined all necessary quantities to perform the theoretical analysis of the dipole transitions from the bound state of the Borromean nucleus ^{9}Be to its continuous spectrum states.

3 Three-cluster photodisintegration ^{9}Be

In this nucleus, the 1/2$^+$ resonance state is a mysterious state which is seen in one set of experiments, but does not observed in other set of experiments. Such a situation is also observed with different theoretical models. It was shown in References [1] and [16] that the AMHHB model confirms the existence of the 1/2$^+$ resonance state. These results were obtained with the modified Hasegawa-Nagata potential [17, 18] in Reference [1] and the Minnesota potential [19, 20] in Reference [16]. Energies of resonance states in ^{9}Be are determined from the three-cluster α+α+n threshold.

In Table 1 we show the input parameters of calculations for ^{9}Be and the energy of the ground state, and energies and widths of the 1/2$^+$ resonance states. These results are obtained in Reference [1] with the MHNP and in Reference [16] with the MP. Both potential creates at least two resonance states, one of which is close to the three-cluster threshold (E=0.248 MeV and E=0.338 MeV) and other lies at energy E=1.664 MeV and E=1.432 MeV. Energies of resonance states obtained with two different nucleon-nucleon potential are close, however their widths are quite different.

In Table 2 we display two-body threshold energies of ^{8}Be(0$^+$)+n and ^{5}He(3/2$^-$)+α, as they play an important role in the photodisintegration of ^{9}Be. These energies are measured from the three-cluster threshold and they include energies of the 0$^+$ resonance state in ^{8}Be and the 3/2$^-$ resonance state in ^{5}He, respectively.

Table 1 – Energy of the ^{9}Be ground state and parameters of the 1/2$^+$ resonance states

Input parameters			$J^\pi=3/2^-$	$J^\pi=1/2^+$			
Potential	b (fm)	u/m	E (MeV)	E (MeV)	Γ (keV)	E(MeV)	Γ (MeV)
MP	1.285	0.9280	-1.555	0.248	15	1.664	1.520
MHNP	1.317	0.4389	-1.574	0.338	168	1.432	0.233

Table 2 – Parameters of resonance states in two-cluster subsystem of ^{9}Be

Channel	^{8}Be(0$^+$)+n		^{5}He(3/2$^-$)+α	
Potential	E (MeV)	Γ (keV)	E (MeV)	Γ (MeV)
MP	0.172	0.748	1.059	1.116
MHNP	0.859	958.40	0.385	0.209

Let us turn our attention to the dipole transitions. In Figure 1 we display the dipole transition probability from the 3/2$^-$ ground state to the 1/2$^+$ continuous spectrum states of^{9}Be. The bar plots indicate energy (the centre of the bar) and with of the 1/2$^+$ resonance states. In Figure 1 we show the dipole transition probability for three wave functions of continuous spectrum state. These functions are distinguished by the entrance channel K = 0, K = 2 and K = 4. As one can see, the first wave function is dominant-channel in the present region of energy. Besides the first 1/2$^+$ resonance state is created in this channel, the dipole transition probability is very small.

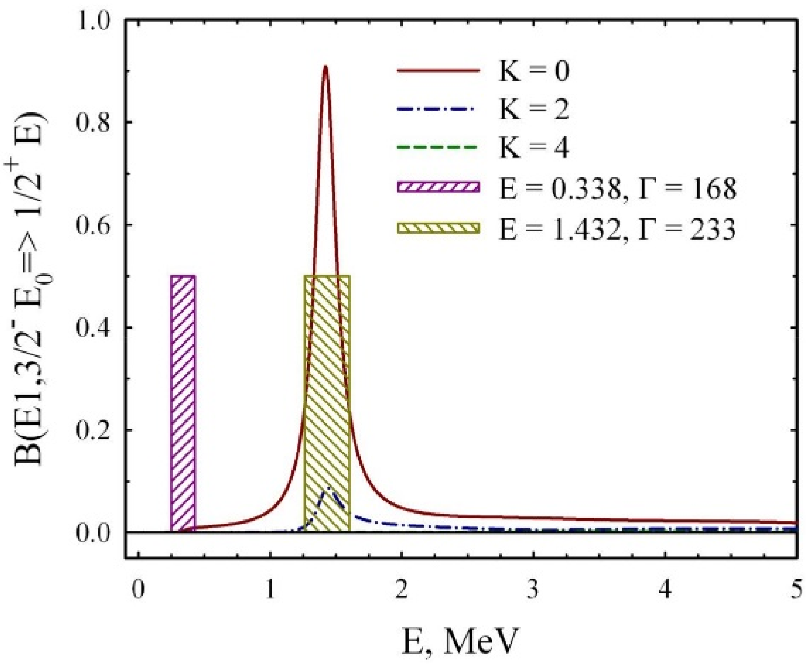

Figure 1 – Distribution of the dipole transition probability over continuous spectrum states of ^{9}Be. The bars present the energy (in MeV) and width (in keV) of the 1/2$^+$ resonance states.

It is interesting to note that shape of the function B(E1; 3/2$^-$ E$_0$=>1/2$^+$E) is similar to the weight of the internal part of the scattering 1/2+ wave function. The latter is displayed in Figure 2 for three dominant wave functions of the 1/2$^+$ state generated by the entrance channels with K = 0, K = 2, and K = 4, respectively.

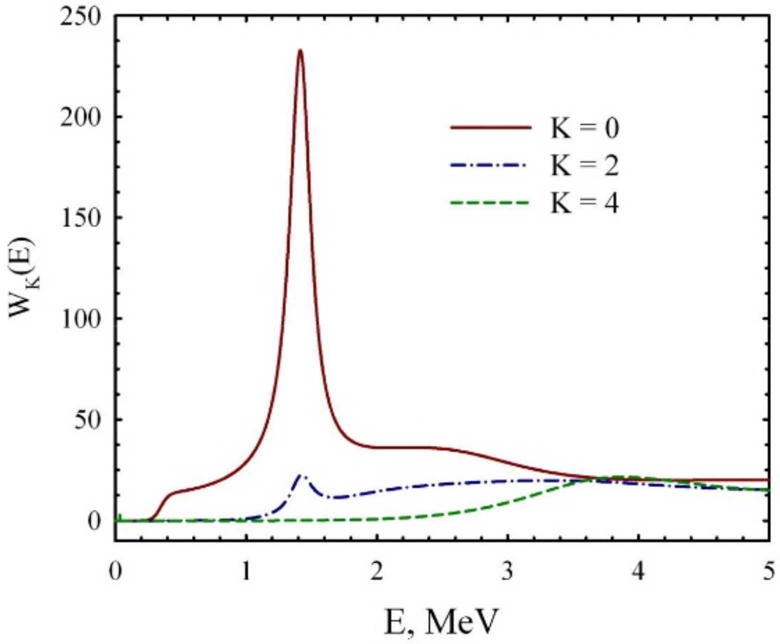

Figure 2 – Weights $W_K(E)$ of the internal part of the many-channel wave function describing three-cluster scattering in the state $J^{\pi} = 1/2^{+}$.

Contribution of the first $1/2^{+}$ resonance state to the dipole transition is shown in Figure 3. One can see that the contribution of the first $1/2^{+}$ resonance state is not as prominent as for the second $1/2^{+}$ resonance state. This is a result of a kinematical factor that suppresses the dipole transition at the low-energy region.

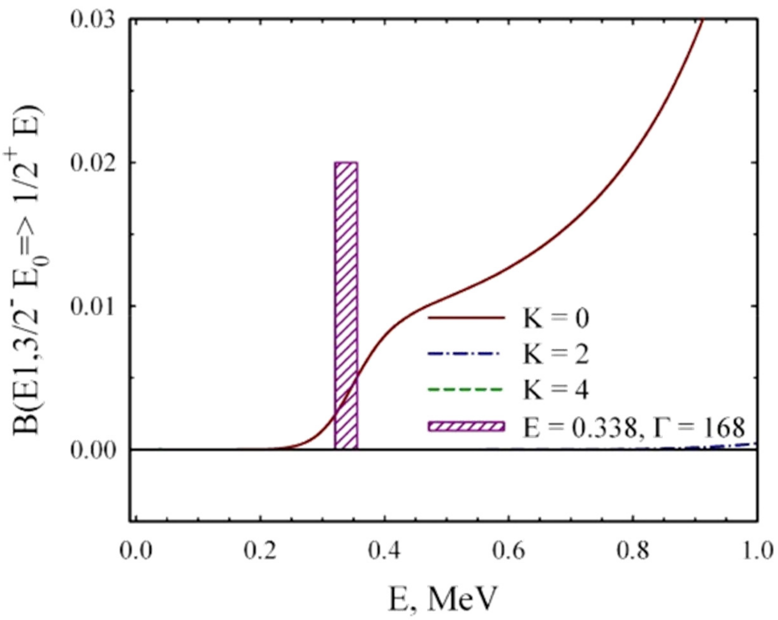

Figure 3 – The distribution of the dipole transition in ^{9}Be around the first $1/2^{+}$ resonance state.

4 Conclusions

We have considered the photodisintegration of the nucleus ^{9}Be. The consideration has been performed within a microscopic three-cluster model α+α+n. The model employs the full set of six-dimension oscillator functions to describe relative motion of clusters. Oscillator functions are numerated the quantum numbers of the hyperspherical harmonics method. The hyperspherical harmonics are very suitable for implementing the boundary conditions for wave functions of three-cluster continuous states. The dipole transitions from the ground $3/2^-$ state to the $1/2^+$ states of three-cluster continuum were studied in detail. We demonstrated that the low-lying $1/2^+$ resonance state weakly contributes to the dipole transition probability, while the second $1/2^+$ resonance state has strong impact on the dipole transition probability.

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