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The first excited $1/2^+$ state in ^{9}Be and ^{9}B

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Nuclear states observed around threshold energies provide us with interesting problems associated with the nuclear cluster structure [1, 2, 3, 4]. The first excited $J^\pi = 1/2^+$ state of ^{9}Be [5], which is an α+ α + *n* Borromean nucleus, is one of the typical examples in light nuclei. This state of ^{9}Be has been observed as a sharp peak above the ^{8}Be + *n* threshold energy in the photo-disintegration cross section of $\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ [6, 7]. The strength of the peak has a strong influence on the reaction rate of the ^{9}Be synthesis. We performed the calculations using an α + α + *n* three-body model [8, 9] and the complex scaling method (CSM), which well reproduces the observed photo-disintegration cross section. However, the result indicates that the $1/2^+$ state shows the *s*-wave virtual-state character of ^{8}Be+*n*. In addition to this problem, we discuss a mirror state problem of the first excite $1/2^+$ state in ^{9}B.

Key words: cluster model, photo-disintegration cross section, virtual state.
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1 Introduction

Nuclear states observed around threshold energies provide us with interesting problems associated with the nuclear cluster structure [1, 2, 3, 4]. Most of them are also interesting astrophysically from the viewpoint of nucleosyntheses. The first excited $J^\pi = 1/2^+$ state in ^{9}Be [5], which is an α + α + *n* Borromean nucleus, is one of the typical examples in light nuclei.

The reaction rate of the ^{4}He(αn, γ)^{9}Be reaction is crucial to understand the productions of heavy elements. In the α(αn, γ)^{9}Be reaction, a sequential process, ^{4}He(α, γ) ^{8}Be(n, γ) ^{9}Be, has been considered as a dominant one. However, owing to the short life-time of the ^{8}Be ground state (~ 10^{-16} s), a direct measurement of the ^{8}Be(*n*, γ)^{9}Be reaction is impossible. For an alternative way, the cross section of its inverse reaction, ^{9}Be(γ, *n*)^{8}Be, has been measured to deduce the cross section of ^{8}Be(*n*, γ)^{9}Be.

The low-lying $1/2^+$ state have a impact on the reaction rate of ^{8}Be(*n*, γ)^{9}Be in stellar environments. This state of ^{9}Be has been observed as a sharp peak above the ^{8}Be + *n* threshold energy in the photo-disintegration cross section of $\gamma + {}^9\text{Be} \rightarrow \alpha + \alpha + n$ [6, 7]. The strength of the peak has a strong influence on the reaction rate of the ^{9}Be synthesis. From a theoretical side, it is interesting to answer how the low-lying $1/2^+$ state of ^{9}Be contributes to the ^{8}Be(*n*, γ)^{9}Be reaction.

We perform the calculations using an α + α + *n* three-body model [8, 9] and the complex scaling method (CSM) [10, 11]. Applying the three-cluster potential, we show that the observed photo-disintegration cross section [6, 7] is well reproduced. And, the result indicates that the $1/2^+$ state shows the s-wave virtual-state character of ^{8}Be + *n*.

In this report, we explain our results of the first excite $1/2^+$ state in ^{9}Be in comparison with those of other previous studies [12 - 17], because it has been a long-standing problem whether the $1/2^+$ state is a resonant or virtual state. In addition to this problem, we discuss a mirror state problem of the first excite $1/2^+$ state in ^{9}B.

In the next section, we will briefly explain the α + α + *n* three-body model [8, 9], and show the results of the photo-disintegration cross section. In Sec. 3, the result of the complex scaling method for the $1/2^+$ state is discussed to show no resonance solutions for ^{9}Be. In Section 4, the $1/2^+$ state in ^{9}B is shown to be obtained as a resonant state, and the comparison of energy levels for ^{9}Be and ^{9}B is discussed. Finally, summary is given in Section 5.

2 Photo-disintegration of ^{9}Be

To understand the origin of a low-energy peak in the photo-disintegration cross section just above the ^{8}Be + *n* breakup threshold energy in ^{9}Be, we investigate the *E*1-transition strength using an α + α + *n* three-body model [8, 9]. The Hamiltonian for the relative motion of the α + α + *n* three-body system for ^{9}Be is given as

$$H = \sum_{i=1}^{3} t_i - T_{cm} + \\ + \sum_{i=1}^{2} V_{\alpha n}(\xi_i) + V_{\alpha\alpha} + V_{PF} + V_3, \qquad (1)$$

where t_i and $T_{\rm cm}$ are kinetic energy operators for each particle and the center of mass of the total system, respectively. The interactions between the neutron and the α particle is given as $V_{\alpha n}(\xi_i)$, where ξ_i is the relative distance between them. We here employ the KKNN potential [18] for $V_{\alpha n}$. For the α-α interaction $V_{\alpha\alpha}$, we employ a folding potential of the effective NN interaction [19] and the Coulomb interaction:

$$V_{\alpha\alpha}(r) = v_0 \exp(-ar^2) + \frac{4e^2}{r}\,\mathrm{erf}(Br), \qquad (2)$$

where $v_0 = -106.09$ MeV, $a = 0.2009$ fm^{-2}, and $\beta = 0.5972$ fm^{-1}. The pseudo-potential $V_{PF} = \lambda|\Phi_{\rm PF}\rangle\langle V_{\rm PF}|$ with $\lambda = 10^6$ MeV is expressed by the projection operator to remove the Pauli forbidden states $\Phi_{\rm PF}$ from the relative motion of α-α and α-n.

In the Hamiltonian of Equation (1), two-cluster potentials $V_{\alpha n}$ and $V_{\alpha\alpha}$ are fixed so as to reproduce the observed scattering data of αn and α-α, respectively. Since the antisymmetrization effects are taken into account by the Pauli-potential $V_{\rm PF}$ but a three-cluster exchange effect is not included explicitly in this calculation, we introduce the phenomenological three-cluster potential V_3 to investigate the photo disintegration of ^{9}Be by reproducing the breakup threshold energy into α +α +*n*. The explicit form of V_3 is given by

$$V_3 = v_3 \exp(-\mu\rho^2), \qquad (3)$$

where ρ is the hyper-radius of the α + α + *n* system. The hyper-radius is defined as

$$\rho^2 = 2r^2 + \frac{8}{9}R^2, \qquad (4)$$

where *r* is the distance between two α-particles and *R* is that between the neutron and the center of mass of the α + α subsystem.

In Figure 1, calculated photo-disintegration cross sections are shown. The dashed and dotted lines are results with and without the three-body potential of $v_3 = 1.10$ MeV and $\mu = 0.02$ fm. The black solid line represents the cross section calculated by using an attractive three-body potential with $v_3 = -1.02$ MeV. The experimental data below $E_\gamma = 2.2$ MeV are taken from References [6, 7]. The arrow indicates the threshold energy of the ^{8}Be(0^+) + *n* channel.

The result calculated with an appropriate strength v3 of the three-cluster interaction well reproduces the cross section peak observed just above the ^{8}Be(0^+) + *n* threshold.

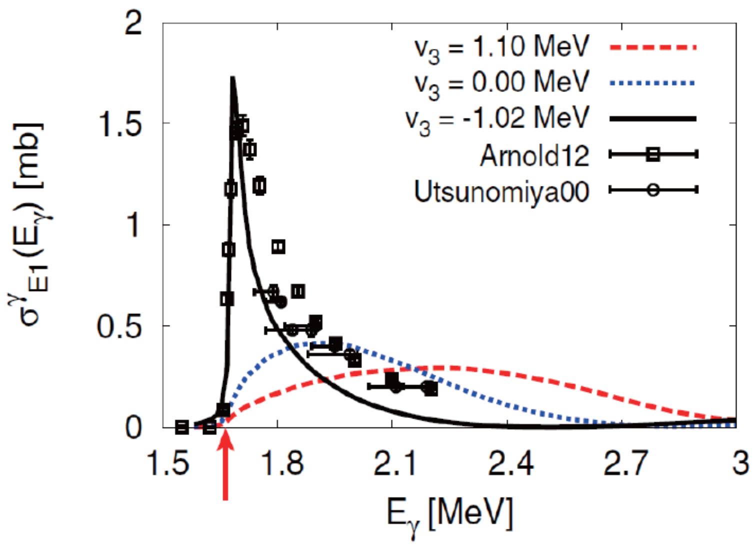

Figure 1 – Calculated photo-disintegration cross sections in comparison with experimental data.

3 The virtual-state property of ^{9}Be $(1/2_1^+)$

For the problem that the first excited $1/2^+$ state in ^{9}Be is resonant or virtual state, we have many studies so far [12 - 17]. To see whether the peak of the photo-disintegration cross section is due to resonances or not, we apply the complex scaling method to the α+ α + n model and search for the $1/2^+$ resonant states. The complex-scaled Schrodinger equation is given as

$$H^{\theta}\Psi_J(\theta) = E_J^{\theta}\Psi_J(\theta), \tag{5}$$

where J is the total spin of the α+α+n system. The complex-scaled Hamiltonian and wave function are

$$H^{\theta} = U(\theta)HU^{-1}(\theta), \qquad \Psi_J(\theta) = U(\theta)\Psi_J, \tag{6}$$

respectively. The complex scaling $U(\theta)$ with a real parameter $0 \leq \theta \leq 45°$ transforms the relative coordinates as

$$U(\theta); \quad r \to re^{i\theta}, \qquad R \to Re^{i\theta}. \tag{7}$$

The calculated eigenvalue distribution of the $1/2^+$ states is shown in Figure 2. The result indicates no resonance solutions for $\theta = 15°$. Although there may exist a resonance solution with a large width, which cannot be solved with $\theta = 15°$, it is not consistent with observed data of the width $\Gamma = 217 \pm 10$ KeV [5]. And we could not find such a resonant state by the analytical continuation for the three-cluster potential strength [8].

On the other hand, we obtain the resonant solution for the $1/2^+$ state in the mirror nucleus ^{9}B, where the same Hamiltonian (Equation (1)) for the α + α + p model with the

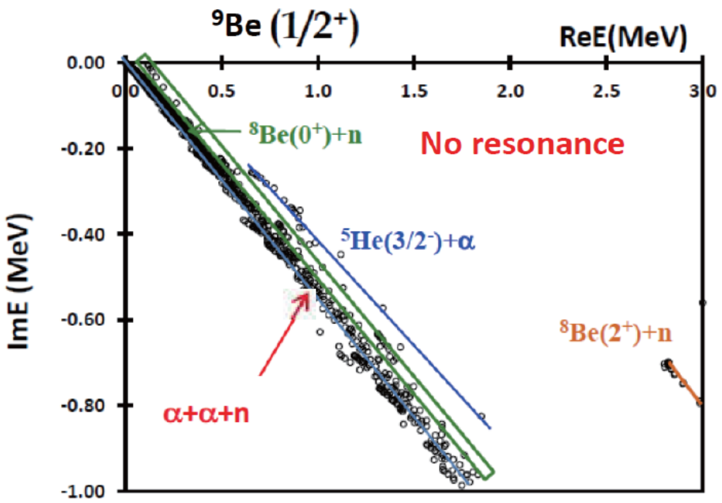

Figure 2 - Energy eigenvalue distribution of $1/2^+$ states of 9Be measured from the α+α+*n* threshold with scaling angle θ = 15°. The solid, dashed, and dotted lines represent the branch cuts for α+α+*n*, ^{8}Be(0^+) + *n*, and ^{5}He(3/2⁻) + α continua, respectively.

Coulomb interaction for the proton *p* are used. In Figure 3, the $1/2^+$ resonant state is shown with a circle. This resonance solution is understood to be reproduced by the Coulomb interaction between the valence proton and two α clusters, which does not exist in the α + α + n system.

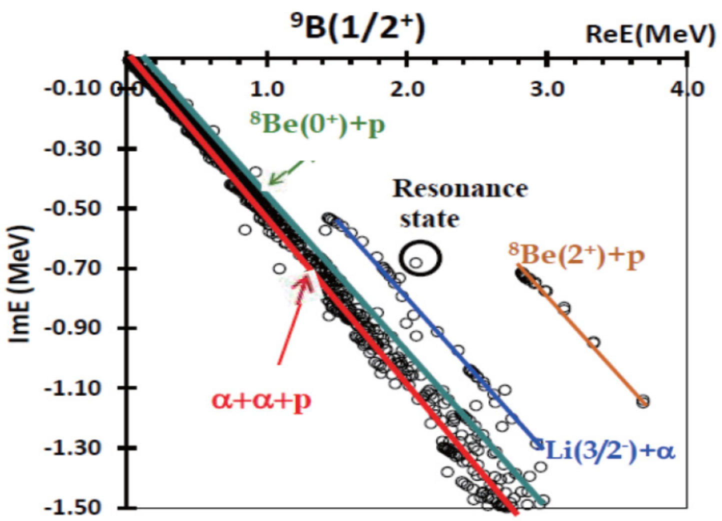

Figure 3 – Energy eigenvalue distribution of 1=2+ states of 9B measured from the α + α + p threshold with scaling angle θ = 15°. The solid, dashed, and dotted lines represent the branch cuts for α + α + p, ^{8}Be(0^+) + *p*, and ^{5}Li(3/2⁻) + α continua, respectively

The virtual state property of the $1/2^+$ state in ^{9}Be was studied in detail by using the ^{8}Be+*n* model [20-30]. It is confirmed that the virtual state of the neutron *s*-wave is embedded in the continuum without a barrier potential. Furthermore, it is shown that we cannot distinguish virtual state from resonant state in the shape of the cross section peak, when the resonance appears at a very small energy from the threshold.

4 Mirror States in ^{9}Be and ^{9}B

In addition to the $1/2^+$ state, low-lying states of ^{9}Be are calculated within the α+ α + n model. The observed photo-disintegration cross sections [6, 7] are shown to be well explained over a wide energy region [9]. The energy levels of ^{9}Be are presented in Figure 4 together with experimental results [5]. The first excited $1/2^+$ state does not have correspondence in the present calculation, and the $3/2_2^-$ state is predicted to be about 1 MeV lower than the experiment. However, other states are well reproduced.

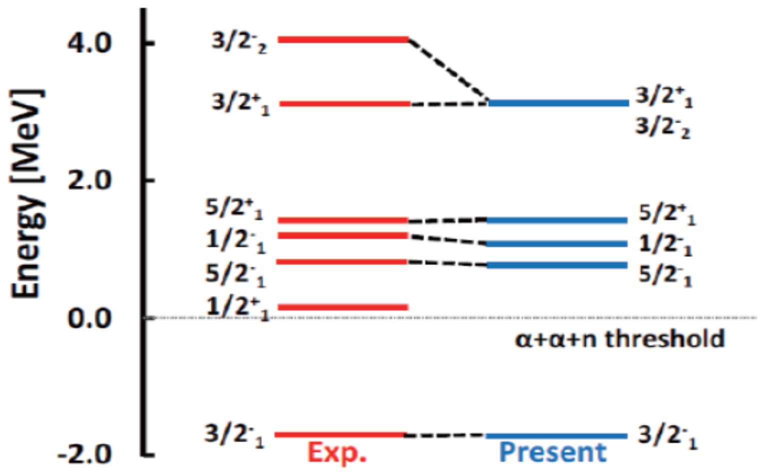

Figure 4 – Energy levels of ^{9}Be. The present calculation is compared with the experimental data taken from Ref.[5]

In Figure 5, we show the present result of energy levels for ^{9}B in comparison with observed data [5]. The low-lying states, which are all resonant states, are well reproduced except for the first excited $1/2^+$ state. The calculated state is rather higher than the experimental one.

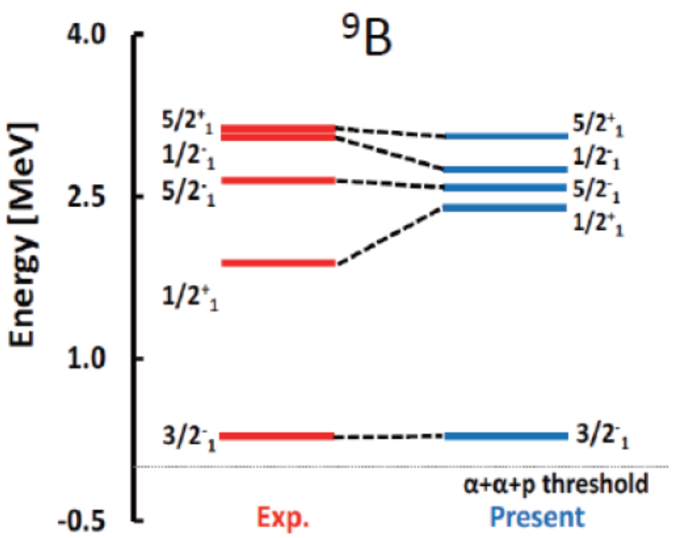

Figure 5 – Energy levels of ^{9}B. The present calculation is compared with the experimental data taken from Ref. [5]

This state of ^{9}B is the mirror of the virtual state in ^{9}Be, which is understood to have the *s*-wave configuration of the neutron around the ^{8}Be (= α+α) core. The Thomas-Ehrman effect [23] suggests that the *s*-wave proton of the mirror nucleus has a weak effect from the Coulomb interaction. Then the energy shift of the *s*-wave proton configuration is expected to be smaller than those of other states. Thus, the present result shows an inverse tendency of the energy relation between the $1/2^{+}$ states in ^{9}Be and ^{9}B.

5 Summary

It has been a long standing problem that the peak of the photo-disintegration cross section observed just above the ^{8}Be+*n* threshold in ^{9}Be causes from the $1/2^{+}$ resonant state or a neuron *s*-wave virtual. The complex scaled α + α + n model shows to reproduces the observed peak of the $1/2^{+}$ state due to a neuron *s*-wave virtual state of ^{8}Be(0^{+}) + *n*. We discussed a mirror state problem of the first excite $1/2^{+}$ state in ^{9}B.

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