UDC533.9

Influence of exchange and shielding on collisional entanglement fidelity in strong quantum recoil semiconductor plasmas

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The electron-exchange and shielding on the entanglement fidelity for the elastic electron-ion collision is investigated in strong quantum recoil semiconductor plasmas. The effective Shukla-Eliasson potential and the partial wave method are employed to obtain the collisional entanglement fidelity as a function of the electron-exchange parameter, collision energy, Fermi energy, and plasmon energy. The result shows that the influence of electron-exchange suppresses the transmission of quantum information in strong quantum recoil semiconductor plasmas. It is also found that the collisional entanglement fidelity decreases with an increase of the Fermi energy. Additionally, the collisional entanglement fidelity increases with increasing plasmon energy.

Keywords: strong quantum recoil semiconductor plasmas, exchange effect, entanglement fidelity.
PACS number(s): 52.20.Fs; 34.80.Bm; 52.20.-j

1 Introduction

Recently, the transmission of quantum information and entanglement fidelity for the scattering process have been of great interests since these processes have shown that the quantum correlations play crucial roles in understanding the quantum measurement process [1-4]. It has been shown that the entanglement fidelity for the collision process in plasmas can provide useful information on the influence of plasma shielding for the collision dynamics since the entanglement fidelity is related the transmission of quantum information [5]. Especially, the collision process in plasmas has received considerable attention in astrophysical and laboratory plasmas since this process has been widely us the plasma diagnostic tool to provide various plasma parameters [6-9]. It has been shown that the conventional weakly coupled classical plasma would be described by the Yukawa-type Debye-Hückel model since the average interaction energy between particles is small compared to the average kinetic energy of a plasma particle in ideal or weakly coupled classical plasmas [10]. Recent years, the physical characteristics and properties of quantum plasmas have been extensively investigated since the quantum plasmas have been found in nano-wires, quantum dot, quantum well, and semiconductor devices as well as in dense astrophysical environments [11-20]. Hence, it would be expected that the screened interaction potential in quantum plasmas is quite different from the standard Debye-Hückel potential in ideal or weakly coupled classical plasmas due to the Bohm potential and quantum statistical effects [14,15,19]. Additionally, a very recent investigation [20] has shown that the influence of electron-exchange due to the electron-1/2 spin plays an important role in the electric potential and plasma dielectric function dielectric function in degenerate quantum plasmas. Then, it can be expected that the collisional entanglement fidelity in degenerate quantum plasmas would be quite different from that in ideal or weakly coupled classical plasmas. However, the entanglement fidelity for the scattering process in degenerate quantum plasmas has not been investigated as yet. Thus, in this paper, we investigate the influence of electron-exchange and quantum screening on the entanglement fidelity for the elastic collision process in degenerate quantum plasmas. The partial wave analysis [21] and the effective Shukla-Eliassonpotential [20] are employed to obtain the entanglement fidelity for the elastic collision process in quantum plasmas as a function of the electron-exchange parameter, Fermi energy, plasmon energy, and collision energy. The variation of the electron-exchange and quantum screening effects on the collisional entanglement fidelity for the elastic electron-ion collision in degenerate quantum plasmas is also discussed.

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2 Calculations

Using the non-relativistic quantum hydrodynamic method [19], the continuity and momentum equations in dense quantum plasmas are represented by, respectively,

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{v})=0, \quad (1)$$

$$m\left(\frac{\partial \mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\,\mathbf{v}\right)=-\frac{1}{n}\nabla P+e\nabla\varphi+\frac{\hbar^2}{2m}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right)+\nabla V_{xc}, \quad (2)$$

where n, $\mathbf{v}$, m, and P are, respectively, the number density, velocity, mass, pressure of the electron, φ is the electric potential, and V_{xc} is the electron-exchange potential. In the momentum equation, Eq. (2), ∇P term is known as the quantum statistical effect owing to the influence of fermionic behavior of the plasma electrons, $\nabla^2\sqrt{n}$ term shows the Bohm potential term corresponding to quantum tunneling and wave packet spreading [18],and $V_{xc}\{=-0.985e^2n^{1/3}[1+(0.034/a_0n^{1/3})\ln(1+18.37a_0n^{1/3})]\}$ term represents the additional potential effect due to the influence of electron-exchange caused by the property of the electron-spin, $a_0(=\hbar^2/me^2)$ is the first Bohr radius of the hydrogen atom, $\hbar$ is the rationalized Planck constant, and $-e$ is the charge of the electron. It has been also shown that the quantum hydrodynamic model would be quite useful to investigate the physical characteristics of the transport process in quantum plasmas. It has been also shown that the hydrodynamic formulation of the Schrödinger-Poisson [18] system is quite convenient to investigate the physical properties and characteristics of quantum plasma systems. Recently, Shukla and Eliasson [20] obtained the plasma dielectric function ε_{SE} in semiconductor quantum plasmas such as

$$\varepsilon_{SE}(k,k_S)=\{1+[(k^2/k_S^2)+\alpha k^4/k_S^4]/[1+(k^2/k_S^2)+\alpha k^4/k_S^4]\}^{-1}$$

including the electron-exchange and quantum shielding effects with the quasi stationary density perturbations, where k is the wave number, $k_s[=\omega_p/(v_F^2/3+v_{ex}^2)^{1/2}]$ is the inverse effective Thomas-Fermi screening length, ω_p is the electron plasma frequency, v_F is the electron Fermi velocity, v_{ex} is the electron-exchange velocity caused by the electron-exchange phenomena, and $\alpha[=\hbar\omega_p^2/4m^2(v_F^2/3+v_{ex}^2)^2]$ is the quantum recoil parameter. It is also found that the effective electric potential $\varphi_{SE}(r)$ of a test change Q in dense quatum plasmas is obtained by $\varphi_{SE}(r)=$ $=(Q/2\pi^2)\int d^3\mathbf{k}\,e^{i\mathbf{k}\cdot\mathbf{r}}/(k^2\varepsilon_{SE})$ using the Shukla-Eliasson plasma dielectric function ε_{SE} when the plasmon energy $E_p(=\hbar\omega_p)$ is comparable or smaller than the Fermi energy $E_F(=mv_F^2/2)$ [20]. Using the effective electric potential model [20,21], the Shukla-Eliasson effective interaction potential $V_{SE}(r,\alpha,k_s)$ between the project tile electron and the dressed ion with nuclear charge Ze in strong quantum recoil semiconductor plasmas, i.e., when $\alpha>1/4$, would be given by

$$V_{SE}(r,\alpha,k_s)=-\frac{Ze^2}{r}\left[\cos(k_ir)+b(\alpha)\sin(k_ir)\right]\exp(-k_rr), \quad (3)$$

where $b(\alpha)=(4\alpha-1)^{-1/2}$ and the effective inverse screening lengths $k_r(\alpha)$ and $k_i(\alpha)$ are given by $k_r(\alpha)=k_s[(4\alpha)^{1/2}+1]^{1/2}/(2\alpha)^{1/2}$ and $k_i(\alpha)=k_s[(4\alpha)^{1/2}-$. $-1]^{1/2}/(2\alpha)^{1/2}$. It is interesting to note that the effective interaction potential shows the oscillatory behavior in semiconductor quantum plasmas. It can be shown that, in the limit $\alpha\to 1/4$, the Shukla-Eliasson effective interaction potential $V_{SE}(r)$ turns out to be the modified Thomas-Fermi screened potential, i.e., $V_{SE}(r)\to V_{TF}(r)=-(Ze^2/r)(1+k_s/\sqrt{2})\times$, $\times e^{-\sqrt{2}k_sr}$ since $k_r\to\sqrt{2}k_s$ and $k_i\to 0$ as $\alpha\to 1/4$.

For an interaction potential $V(\mathbf{r})$, the stationary-state non-relativistic Schrödinger equation [22] can be written as

$$\left(\nabla^2+k^2\right)\varphi(\mathbf{r},k)=\frac{2\mu}{\hbar^2}V(\mathbf{r})\varphi(\mathbf{r},k), \quad (4)$$

where $k(=\sqrt{2\mu E/\hbar^2})$ is the wave number, μ is the reduced mass of the collision system, $E(=\mu v^2/2)$ is the collision energy, v is the collision velocity, and $\varphi(\mathbf{r},k)$ is the solution of the scattered wave equation. The final state wave function $\varphi(\mathbf{r},k)$ would be then represented by the partial-wave expansion method [21] for the angular momentum quantum number l:

$$\varphi(\mathbf{r},k)=(2\pi)^{-3/2}\sum_{l=0}^{\infty}i^l(2l+1)A_l(k)\,R_{k,l}(r)P_l(\cos\theta), \quad (5)$$

where i is the pure imaginary number, $A_l(k)$ is the expansion coefficient, $P_l(\cos\theta)$ is the Legendre polynomial with order l, and $R_{k,l}(r)$ is the solution of the radial wave equation. For a spherically symmetric potential $V(r)$, the expansion coefficient [22] $A_l(k)$ would be represented in the following integral form:

$$A_l(k) = \left[1+2i\frac{\mu}{\hbar^2}k\int_0^\infty dr\, r^2 j_l(kr)R_{k,l}(r)V(r)\right]^{-1}, \quad (6)$$

and the equation for the radial wave function $R_{k,l}(r)$ is given by

$$\left[\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d}{dr}\right)-\frac{l(l+1)}{r^2}-\frac{2\mu}{\hbar^2}V(r)+k^2\right]R_{k,l}(r)=0, \quad (7)$$

where $j_l(kr)$ is the spherical Bessel function with order l. Hence, the solution $R_{k,l}(r)$ of the radial wave equation would be obtained by

$$R_{k,l}(r) = j_l(kr)+\frac{2\mu}{\hbar^2}k\left[\int_0^r dr'\, r'^2 j_l(kr')\,n_l(kr)\,R_{k,l}(r')V(r') + \int_r^\infty dr'\, r'^2\, j_l(kr)\,n_l(kr')\,R_{k,l}(r')V(r')\right]. \quad (8)$$

where $n_l(kr)$ is the spherical Neumann function with order l.

In recent years, the entanglement fidelity for the scattering process has been extensively investigated in various plasmas since it has shown that the quantum correlation and plasma screening effects play crucial roles in the quantum measurement process [2,5]. A recent investigation [1] has shown that the scattering entanglement fidelity $F(k)$ for the collision process would be obtained by the absolute square of the scattered wave function $\varphi(\mathbf{r},k)$ for a given interaction potential. Since it is obvious that the partial s-wave $(l=0)$ provides the main contribution to low-energy atomic collisions, the scattering entanglement fidelity would be obtained by the expansion coefficient $A_l(k)$ and the radial wave equation $R_{k,l}(r)$ in the following form:

$$F(k) \propto \frac{\left|\int_0^\infty dr\, r^2 R_{k,0}(r)\right|^2}{1+\left|\frac{2\mu}{\hbar^2}k\int_0^\infty dr\, r^2 R_{k,0}(r)V(r)\right|^2}. \quad (9)$$

Hence, it would be expected that the influence of electron-exchange and quantum screening on the electron-ion collisional entanglement fidelity in strong quantum recoil semiconductor plasmas would be expressed by the fidelity ratio function $R_{ex}(k,\alpha,k_s)[\equiv F_{SE}(k,\alpha,k_s)/F_C(k)]$ for the low-energy electron-ion collision between then tanglement fidelities for the Shukla-Eliasson effective potential $V_{SE}(r,\alpha,k_s)$ and a pure Coulomb case $(V_C=-Ze^2/r)$:

$$R_{ex}(k,\alpha,k_s) = \left[1+\left|\frac{2Ze^2\mu}{\hbar^2}k\int_0^\infty dr\, r\frac{\sin(kr)}{kr}\right|^2\right] \Big/ \left\{1+\left|\frac{2Ze^2\mu}{\hbar^2}k\int_0^\infty dr\, r\left[\cos(k_r(\alpha)r)\exp\left(-k_r(\alpha)r\right) + b(\alpha)\sin(k_i(\alpha)r)\exp\left(-k_r(\alpha)r\right)\right]\right|^2\right\}, \quad (10)$$

where the influence of electron-exchange and quantum screening on the entanglement fidelity would be explored through the strong quantum recoil parameter α as well as the effective inverse screening lengths k_r and k_i. After some mathematical manipulations using $V_{SE}(r)$, the Shukla-Eliasson entanglement fidelity ratio function $R_{ex}(\bar{E},\bar{E}_F,\bar{E}_P,\beta)$ including the influence of electron-exchange and quantum screening in strong quantum recoil semiconductor plasmas is then found to be

$$R_{ex}(\bar{E},\bar{E}_F,\bar{E}_P,\beta) = \frac{1+4\left(\frac{m_*}{m}\right)^2\frac{1}{\bar{E}}}{1+4\left(\frac{m_*}{m}\right)^2\frac{1}{\bar{E}}G_{ex}(\bar{E},\bar{E}_F,\bar{E}_P,\beta)}. \quad (12)$$

Here, $G_{ex}(\bar{E},\bar{E}_F,\bar{E}_P,\beta)$ is the electron-exchange characteristic function:

$$G_{ex}(\bar{E},\bar{E}_F,\bar{E}_P,\beta)=\left\{\bar{E}\left[\bar{E}-\frac{\bar{k}_s^2(\bar{E}_F,\bar{E}_P,\beta)}{\sqrt{\alpha(\bar{E}_F,\bar{E}_P,\beta)}}\right]+\right.$$

$$+\frac{\bar{E}\sqrt{4\alpha(\bar{E}_F,\bar{E}_P,\beta)-1}}{2\alpha(\bar{E}_F,\bar{E}_P,\beta)}\bar{k}_s^2(\bar{E}_F,\bar{E}_P,\beta)\Bigg/$$

$$\Bigg/\left[\bar{E}-\frac{\bar{k}_s^2(\bar{E}_F,\bar{E}_P,\beta)}{\sqrt{\alpha(\bar{E}_F,\bar{E}_P,\beta)}}\right]^2+$$

$$+\left[\left(\frac{4\alpha(\bar{E}_F,\bar{E}_P,\beta)-1}{2\alpha(\bar{E}_F,\bar{E}_P,\beta)}\right)\bar{k}_s^2(\bar{E}_F,\bar{E}_P,\beta)\right]^2 \quad (13)$$

where m_* is then effective electron mass, $\bar{E}(\equiv \mu v^2/2Z^2Ry)$ is the scaled collision energy, $Ry(=me^4/2\hbar^2\approx 13.6\,\mathrm{eV})$ is the Rydberg constant, $\bar{E}_F(\equiv E_F/Z^2Ry)$ is the scaled Fermi energy, $\bar{E}_p(\equiv E_p/Z^2Ry)$ is the scaled plasmon energy, $\beta(\equiv v_{ex}/v_F)$ represents the electron-exchange parameter, $\alpha(\bar{E}_F,\bar{E}_P,\beta)=(3\bar{E}_P/4\bar{E}_F)^2(1+3\beta^2)^{-2}$, and scaled inverse effective Thomas-Fermi screening length is given by
$\bar{k}_s(\bar{E}_F,\bar{E}_P,\beta)(\equiv \bar{k}_s a_Z)=[(m_*/m)(3\bar{E}_P^2/4\bar{E}_F)/(1+3\beta^2)]^{1/2}$

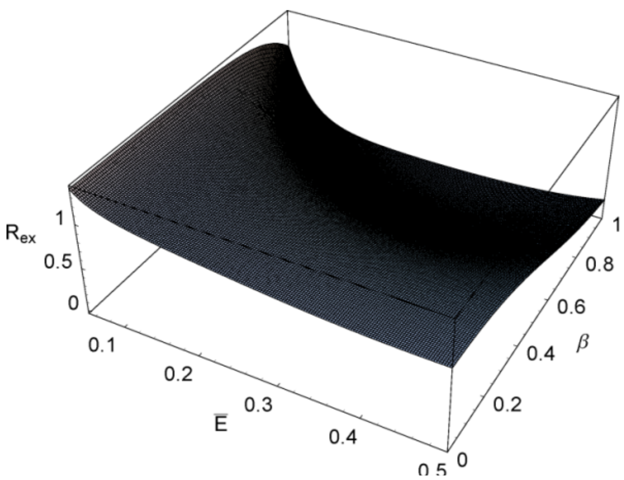

Figure 1 – The surface plot of the entanglement fidelity ratio R_{ex} including the influence of electron-exchange and quantum screening as a function of the scaled collision energy $\bar{E}$ and electron-exchange parameter β when $\bar{E}_F=0.1$ and $\bar{E}_P=0.5$.

From this expression, it can be expected that the electron-exchange effect reduces the effective Thomas-Fermi wave number in quantum plasmas. Recently, the electron-exchange effects on the entanglement fidelity for the electron-ion collision are investigated in weak quantum recoil plasmas [23]. However, in strong quantum recoil cases, the influence of electron-exchange and quantum screening on the electron-ion collisional entanglement fidelity has not been investigated in semiconductor plasmas. Hence, the expression of the Shukla-Eliasson entanglement fidelity ratio function $R_{ex}(\bar{E},\bar{E}_F,\bar{E}_P,\beta)$ [Eq. (13)] would be quite useful to explore the collisional entanglement fidelity in strong quantum recoil semiconductor plasmas including the electron-exchange and quantum screening effects. Additionally, the diffusion coefficient [24] has been obtained by the Green-Kubo relation in dense complex plasma since the diffusion process is one of the most important transport processes in various plasmas. Hence, the investigation of the influence of electron-exchange and quantum screening on the diffusion process in semiconductor quantum plasmas will then be treated elsewhere since the physical information of the diffusion coefficient would be useful for investigating transport processes in plasmas.

3 Main results

In order to specifically investigate the electron-exchange effect on the collisional entanglement fidelity in strong quantum recoil semiconductor plasmas, we consider a case of $m_*=0.07m$ for the semiconductor GaAs quantum well. Figure 1 shows the surface plot of the Shukla-Eliasson collisional entanglement fidelity ratio R_{ex} including the electron-exchange and quantum screening effects as a function of the scaled collision energy $\bar{E}$ and the electron-exchange parameter β in strong quantum recoil semiconductor plasmas. As shown in this figure, the electron-exchange effect suppresses the collisional entanglement fidelity R_{ex} in strong quantum recoil semiconductor plasmas. Hence, we have found that the influence of electron-exchange diminishes the transmission of quantum information in semiconductor plasmas. It is also found that the electron-exchange effect on the collisional entanglement fidelity R_{SE} increases with an increase of the collision energy. Figure 2represents the Shukla-Eliasson entanglement fidelity ratio R_{ex} as a function of the scaled Fermi energy $\bar{E}_F$ for various values of the electron-exchange parameter β. A shown in Fig. 2, the entanglement fidelity decreases with an increase of the Fermi energy in strong quantum recoil semiconductor plasmas. It is also found that the electron-exchange effect on the collisional entanglement fidelity R_{ex} increases with increasing Fermi energy $\bar{E}_F$. Figure 3shows the entanglement fidelity ratio R_{ex} as a function of the scaled plasmon energy $\bar{E}_P$ for various values of the electron-exchange parameter β. As it is seen, the Shukla-Eliasson collisional entanglement fidelity R_{ex} increases with an increase of the plasmon energy $\bar{E}_P$ in strong quantum recoil semiconductor plasmas. From this figure, we have also found that the influence of electron-exchange on the collisional entanglement fidelity R_{ex} is more significant in intermediate plasmon energy regions.

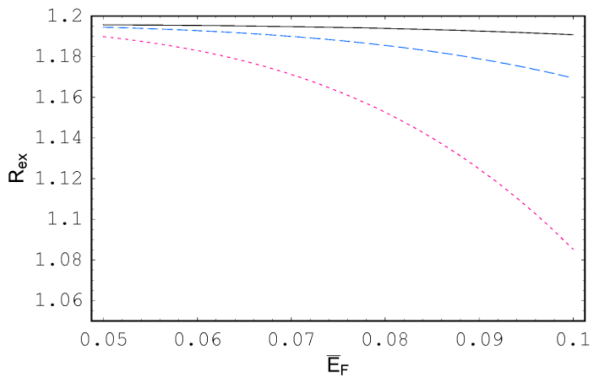

Figure 2 – The Shukla-Eliasson entanglement fidelity ratio R_{ex} as a function of the scaled Fermi energy $\bar{E}_F$ when $\bar{E}=0.1$ and $\bar{E}_P=0.5$. The solid line represents the case of $\beta=0$, i.e., without the electron-exchange effect. The dashed line represents the case of $\beta=0.4$. The dotted line represents the case of $\beta=0.6$.

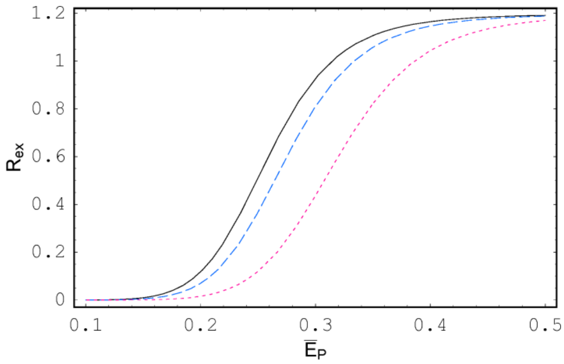

Figure 3 – The Shukla-Eliasson entanglementfidelity ratio R_{ex} as a function of the scaled plasmon energy $\bar{E}_P$ when $\bar{E}=0.1$ and $\bar{E}_F=0.1$. The solid line represents the case of $\beta=0$, i.e., without the electron-exchange effect. The dashed line represents the case of $\beta=0.2$. The dotted line represents the case of $\beta=0.4$.

4 Conclusions

From obtained results, we have found that the electron exchange and plasma screening effects play significant roles on the collisional entanglement fidelity in strong quantum recoil semiconductor plasmas when $\alpha>1/4$. These results would provide useful information on the transmission of quantum information and collisional entanglement fidelity in strong quantum recoil semiconductor plasmas.

Acknowledgments

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (Grant No. 2012-001493).

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