UDC539.171

Model of ^{9}Be nucleus as "quasi-molecular" state of "n+a+a" system

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The ^{9}Be nuclide has been considered as a system of two alpha-particles and one neutron and it has been shown that such system can exist in "quasi-molecular" state, when the wave number of a pair of heavy particles becomes imaginary *q=ik* with the wave number of the light particle being a real quantity. That is, rescattering of light particles on the two heavy particles creates additional attraction between the heavy particles and "binds" this heavy pair. The total energy of the system becomes negative: $E = p^2/m + q^2/M < 0$, m is the mass of the light particle, and M is the mass of the heavy particle. Since *k = k (p)*, the total energy of the system *E(p)* has a minimum for the variable *p*. The estimates were obtained in the Born-Oppenheimer approximation, where the choice of the pair potentials in separable form allows to solve the three body problem in a simple and compact form.

Keywords: beryllium 9, nuclear models, quasi-molecular state, Born-Oppenheimer approximation,three body problem.
PACS number(s): 21.60.-n, 24.10.Cn

1 Introduction

Study and theoretical description of the ^{9}Be nucleus properties still remains one of the urgent problems in nuclear physics [1,2]. Such interest is conditioned by specific properties of this nucleus and its promising use as an effective neutron reflector. (Be-9 stands right after the first unstable nuclei with atomic number A = 8). Its reflection capability has pre-determined its widespread use in nuclear engineering [3,4].

It is known that the light nuclei are good neutron moderators and some of them are good neutron reflectors. Reflectors of thermal and intermediate neutrons are usually made from a substance of moderators used in nuclear reactors [3,4]. In heavy water reactors graphite is used as a reflector due to its availability and good diffusion properties. In light water reactors there is always a layer of water (10 cm or more) as a moderator between active zone and shell of the reactor. This layer is already a reflector and reduces the active zone size.

Intermediate reactor contains some moderator and neutrons absorbed by substance before becoming heat. The best reflector for neutrons is beryllium. Also, beryllium is the best moderator for intermediate reactors of small critical dimensions, i.e. for the reactors with high concentration of fissile material in the active zone. Ordinary water is inferior to beryllium because at energies above 0.1 MeV fast neutrons pass through water easier, than through beryllium [3,4]. Obviously, the reflective properties of beryllium material are directly related to its crystalline structure.

Very often the Be9 nuclide is considered as a system of three particles, i.e. as "n+α+α" system that exists in a form of bound state. And usually the system is analyzed in area of discrete spectrum of pair subsystems. On this way there are some difficulties, where the main problem is the heavy reliance on the parameters of the alpha-alpha potential which has a deep hole at small distances [5]. Opposite, na-potentials does not give any bound states. It means that only the process of rescattering of neutron on two alpha particles can lead to appearance of bound states in the three-body system.

Here the ^{9}Be nuclide has been considered also as the system of two alpha-particles and one neutron. However, it has been shown that such system can exist in "quasi-molecular" state, when the wave number of a pair of heavy particles becomes imaginary $q = i\kappa$ with the wave number of the light particle being a real quantity. That is, rescattering of light particles on the two heavy particles creates additional attraction between the heavy particles and "binds" this heavy pair. The total energy of the system becomes negative: $E = p^2/m + q^2/M < 0$, *m* is the mass of the light particle, and *M* is the mass of the heavy particle. Since *k = k (p)*, the total energy of the system *E(p)* has a minimum for the variable *p*. This model has been considered in the Born-

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Oppenheimer approximation, where the choice of the pair potentials in separable form allows to solve the three body problem in a simple and compact form.

It is shown that the additional attraction between alpha particles increases their usual pair interaction and, therefore, shifts the corresponding pole of the t-matrix from the resonance region to the area of bound states. Interaction of an additional external neutron with such three-body system could lead to displacement in the energy of the primary neutron along the curve *E(p)* from the minimum point. If the strength of such disturbance is small for the dissociation of the three-body system, the primary neutron returns to the minimum point, throwing back the external neutron acting as an elastic wall. As we assume, this is how the reflection effect of neutrons works for the case of beryllium nucleus.

2 Three body model of the nucleus ^{9}Be

Let we analyze and calculate for a system consisting of two alpha particles and one neutron. Mathematically rigorous solution of the three-body problem was given by L.D. Faddeev [6]. The system of Faddeev equations for T-matrix elements can be written as:

$$T_{ij}(Z) = t_i \cdot \delta_{ij} + t_i G_0(Z) \sum_{l \neq i} T_{lj}(Z)\,; \quad i,j = 1,2,3 \qquad (1)$$

where $t_i = V_i + V_i G_0 t_i$, V_i are the pair interaction potentials, and t_i corresponding pair t-matrices. Indices *i, j* denote the number of the pairs, G_0 is a Green function for three free particles. The total T-matrix corresponds to the sum $T = \sum T_{ij}$.

To solve the task one needs to determine the pair *t*-matrices t_i, i.e. there is no need to deal with complex pair interaction potentials for the particles; this makes our analysis task easier. Within the considered model we would first consider the main peculiarities of the pair amplitudes at low energies. Such peculiarities are their resonances: for α,α-subsystem this is a very narrow resonance in *S*-wave at $E_{R,\alpha\alpha} \approx 91.6\,keV$ and width $\Gamma_{\alpha\alpha} \approx 6\,eV$ [5], and for (α+n) – subsystem – a resonance in *P*-wave at $E_R = 0.9 MeV$ and width $\Gamma = 0.6 MeV$ [7].

Let us determine the pair *t*-matrixes that can generate such resonances. This can be done using simple separable potentials $V_i = \bar{\nu}_i(\vec{q}) \cdot \lambda_i \cdot \nu_i(\vec{q}\,')$, where λ_i – is the coupling constant. Then $t_i = \bar{\nu}_i(\vec{q}) \cdot \eta_i(E) \cdot \nu_i(\vec{q}\,')$, where

$$\eta_i^{-1}(E) = \lambda_i^{-1} + I(E), \qquad I(E) = -\int d\vec{q}\, \frac{\nu_i^2(q)}{E - E_s + i\gamma} \qquad (2)$$

$E_s = q_i^2 / 2\mu$, μ - reduced mass in the *i*-subsystem, $E = q_{i0}^2 / 2\mu$. The condition $\eta_i^{-1} = 0$ determines location of pole for the pair scattering amplitude in the complex energy plane and, correspondingly, complex wave numbers.

For the α,α-subsystem we choose

$$\nu_i(\vec{q}) = \nu_S(\vec{q}) = = N_{\alpha\alpha} t (t^2+1)^{-3/2} Y_{00}(\hat{\vec{q}}),$$

where $t = q/\beta_S$. The norm constant $N_{\alpha\alpha}^2 = = 32\pi/(2\mu_{\alpha\alpha}\beta_S)$ corresponds to the normalization $I(E=0)=1$, so that $I(E) = I_S(q_0) = (1-3i\,t_0)/(1-it_0)^3$. Dimensionless coupling constant $\lambda_S = -1+\alpha$, $\alpha = 5.78*10^{-10}$ and the parameter $\beta_S = 6.77\cdot 10^3\,fm^{-1}$ correspond to the experimentally obtained resonance characteristics [6]. One should note that resonance in the $\alpha\alpha$-subsystem satisfies the condition: $x^3 + 3\lambda_S x - 2\lambda_S = 0$, where $x = 1 - i\cdot t_0$.

For the (α+*n*) – subsystem the potential form-factor can be written as $\nu_i(\vec{p}) = \nu_P(\vec{p}) = = N_{n\alpha} t (1+t^2)^{-1} Y_{LM}(\hat{\vec{p}})$, where $t \equiv p/\beta_P$. So, we get $I(E) = I_P(p_0) = (1-2i\,t_0)/(1-it_0)^2$, where $t_0 = p_0/\beta_P$. The condition $\eta_P^{-1} = 0$ brings us to

$$\eta_P^{-1}(E) = \frac{1}{\lambda_P} + \frac{1-2it_0}{(1-it_0)^2} = \frac{1+\lambda_P - 2it_0\cdot(1+\lambda_P) - t_0^2}{\lambda_P\cdot(1-it_0)^2} = 0 \qquad (3)$$

Introducing $t_0 = t_R + it_I$ we get for the case of resonance conditions $\lambda_P = -(1+t_I)$ and $t_R^2 = -t_I(1+t_I)$. As it should be, two poles symmetrical with respect to an imaginary axis $t_R = \pm\sqrt{-t_I(1+t_I)}$ correspond to the quasi-stationary state with energy *E=E$_R$–i Γ/2*. For the (α+*n*) – resonance with energy $E_R = 0.9 MeV$ and width *Γ=0.6MeV* we get: $t_R = \pm 0.17$, $t_I = -0.03$, $\lambda_P = -0.97$, $\beta_P = 1.25\,fm^{-1}$.

3 Solution for the three interacting particles.

Separating in (1) the connected term P_{ij} of amplitude with the relation $T_{ij} = t_i \cdot \delta_{ij} + + |\nu_i> \eta_i P_{ij} \lambda_j < \nu_j|$ one can get the system of equations $P_{ij} = \Lambda_{ij} + \sum_l \Lambda_{il}\,\eta_l\,P_{lj}$. Here $\Lambda_{ij} = = <\nu_i|G_0(Z)|\nu_j>$, $i \neq j$. Then, it follows for the elastic channel (i->i) [8]:

$$P_{ii} = V_{ii}^{ef} + \sum_{i_s} V_{ii_s}^{ef}\, \eta_{i_s}\, P_{i_s i}\,, \qquad (4)$$

$$V_{ii}^{ef} = \sum_{l,k} \Lambda_{il}\, \eta_l (I \cdot \eta^{-1} - \Lambda)_{lk}^{-1}\, \eta_k \Lambda_{ki}, \qquad l,k \neq i\ , \qquad (5)$$

The equations (4) and (5) represent a closed system of equations and are basic for the effective potential method in the three-body problem. Moreover, employing the limit $\xi = m/M \to 0$, where m - mass of a light particle, M – mass of a heavy particle, simplifies determination of the solutions. It is important that all polar peculiarities of the pair scattering amplitudes are preserved. If we accept the solution obtained within such approximation as a basic one, then corrections to it can be made based on conventional perturbation theory. The method is described in details in [8-10]. Below we present the simplified solution of the task.

The calculations show that the ^{9}Be ground state energy depends mainly on the pair resonance parameters and less depends on interactions in the non-resonant regions. The considered above (n+α)-resonance provides us with the value for bound energy of the (α+α+n)-system: E = - *1.285 MeV*. The experimentally determined value is E = - *1.665 MeV* [7]. Still, the additional (n+α) resonance with energy E_R = *4.6 MeV* and width Γ = *4 MeV* [7] amplifies the bound energy up to E= - *1.775 MeV*. But at the following parameters of this resonance E_R = *2.0 MeV*, and Γ = *5.0 MeV* [11], total bound energy is even higher: E = - *2.366 MeV*.

4 Conclusions

Considered above configuration of the system of three particles is, quite possibly, not typical for bound nuclear systems, but it fits the general scattering theory for three particles. One can say that (α+α+n)-system represents a "quasi-molecular" state where the wave number of a pair of heavy particles becomes imaginary $q = i\kappa$, and the wave number of a light particle p remains real. This happens because multiple scattering of a light particle at two heavy ones creates additional attraction between these particles which "couples" these heavy particles. Non-linear dependence $\kappa = \kappa(p)$ determines the coupling strength of heavy particles as a function of the light particle wave number. Total energy of the system $E = p^2/2m - \kappa^2/M$ becomes negative only at certain range of p values and shows a minimum over this variable (see Fig.1). Such state is known in the scattering theory of few particles as a bound state of a subsystem recessed within a continuous spectrum of an adjacent subsystem. Possibly, namely this state pre-determines the uniqueness ^{9}Be of nucleus as a neutron reflector.

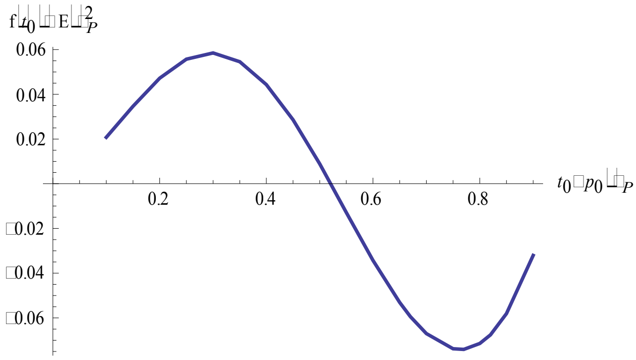

Figure 1 – $E/\beta_P^2 = f(t_0)$ curve.

An additional external neutron scattered at such "flexible" coupled system can result in a shift of initial neutron energy along the curve *E(p)* away from the maximum point. If the disturbing force is low to decouple the three-particle system, the initial neutron returns back to the minimum point and kicks off the external neutron like an elastic wall. Moreover, Pauli Exclusion Principle can be applicable here preventing overlapping of the wave functions from identical neutrons what would also generate additional repulsion.

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