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Mathematical model analysis of nonlinear control systems of power turbines operating in condensation mode

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The work is dedicated to modeling&turbine control systems and studying the stability of a nonlinear system. The dynamics of the turbine regulation system is described by a nonlinear&system of four differential equations. This system of equations describes the mathematical model of a turbine operating in condensing mode. Using the transformation&way, the mathematical model of the steam turbineregulation system is reduced to the conclusion of a problem about the unconditional stability of a nonlinear stationary system of indirect control. In the study of a nonlinear system, the Lyapunov function was applied and the circumstances of unconditional stability were obtained. The conclusion of differential equations systems is executed in vector-matrix form. The results&obtained for a nonlinear system are&used to study the durability of a steam turbine. Numerical calculations are presented that illustrate the probability of using the proposed layout and studying the stability of nonlinear control systems for power turbines.

Key words: stability of a nonlinear system, differential equation, Lyapunov function, mathematical model of a turbine, asymptotic stability, characteristic of nonlinearity.
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Introduction

Mathematic always_ have a special meaning_ in the study of nonlinear power_ turbine regulation systems. They form the basis of analytical ways_ and become an& obligatory part of experimental studies. At the same time, the construction of analytical_ models is considered a furor message that correctly reflects the dynamic qualities of the components [1, 2]. The mathematical model &makes it possible to predetermine the quantitative characteristics of the turbine_ control properties, in fact, which is considered a serious task. Knowledge of these characteristics is necessary when operating turbine regulation frames and when developing new regulation systems. Analytical modeling_ is based on the ways &of the doctrine of automation in the study of turbine regulation frames [3]. If at the beginning of the development of turbine regulation frames &they were limited to studying_ the durability and performance of a linear model, then the test of nonlinear models became generalized. In the leading one, this is due to the complication of claims &to the quality of the turbine regulation frames functioning [4,5].

This paper is devoted to the simulation_ of turbine regulation frames and the study of the stability of the nonlinear model.

Mathematical modeling of turbine control systems.

Let us consider the block diagram shown in Figure 1 of the control system of a turbine operating in the condensation mode [2].

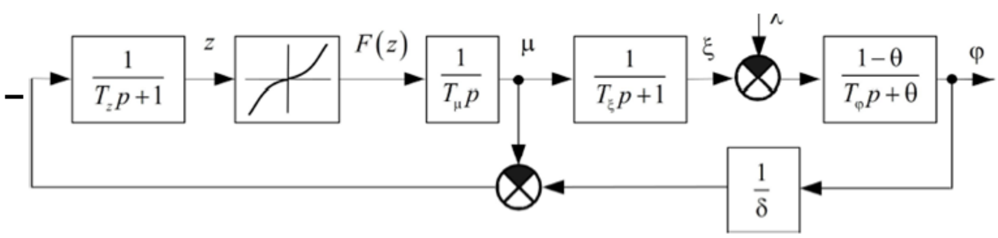

Figure 1 – Block diagram of the turbine control system.

The following notation in Figure 1: z – a relative deviation of the piston valve, λ – the relative deviation of the electric generator load, δ – the degree of non-uniformity of the system of regulation, ξ – the relative change of &steam flow in the turbine, θ – the coefficient of self – regulation, φ – the relative change of the turbine rotor's rotational speed, μ – the relative &deviation of the servomotor's piston, $T_z, T_\mu, T_\varphi, T_\xi$ – time constants.

For small deviations of the spool z from the average position in the control system the nonlinear element F(z)is represented as [2]:

$$F(z) = \begin{cases} 0.5z + 0.25\frac{z^2}{\Delta z_0} signz, by|z| \leq \Delta z_0, \\ z - 0.25\Delta z_0 signz, by|z| > \Delta z_0, \end{cases} \quad (1)$$

where Δz_0 are the relative heights of the triangular profile cut-off spool piston's edges.

According to this control law, a control action is formed with non-linearity F(z), which is fed to the electrohydraulic drive (EHD). In turn, the EHD amplifies the regulation signal in terms of power and converts it into the movement of the servomotor μ [6].

The& mathematical &model of the drive has the form of a differential &equations system (DES):

$$T_\mu \frac{d\mu}{dt} = F(z) \quad (2)$$

The servomotor moves the steam supply valve to the steam turbine [7]. Mathematical model of the steam path:

$$T_\xi \frac{d\xi}{dt} = -\xi + \mu \quad (3)$$

Differential equation of the rotor motion:

$$T_\varphi \frac{d\varphi}{dt} = -\theta\varphi + (1-\theta)(\xi - \lambda) \quad (4)$$

Condensing turbine control system:

$$T_z \frac{dz}{dt} = \left(-\frac{\varphi}{\delta} - \mu\right) - z \quad (5)$$

The dynamics of the turbine control system is described by a nonlinear system of four differential equations [8]. This&system of equations (1)-(5) describes a mathematical model of a turbine operating in the condensation mode.

The turbine control stability

Let's write the equations (2)-(5) in the nonlinear system form:

$$\dot{y_1} = -a_1 y_1 + a_2 y_2 \quad (6)$$

$$\dot{y_2} = -a_3 y_2 + a_4 \varphi(\sigma) \quad (7)$$

$$\dot{y_3} = -a_5 y_1 - a_6 y_3 - a_7 \varphi(\sigma) \quad (8)$$

$$\dot{\sigma} = y_3, \quad (9)$$

where is used the following notations:

$$a_1 = \frac{\theta}{T_\varphi}, a_2 = \frac{(1-\theta)}{T_\varphi}, \\ a_3 = \frac{1}{T_\xi},\ a_4 = \frac{1}{T_\xi T_\mu}, a_5 = \frac{1}{T_z \delta}, \\ a_6 = \frac{1}{T_z},\ a_7 = \frac{1}{T_z T_\mu} \quad (10)$$

$$y_1 = (-\theta\varphi + (1-\theta)(\xi - \lambda))/T_\varphi, \quad (11)$$

$$y_2 = (-\xi + \mu)/\, T_\xi, \quad (12)$$

$$y_3 = \frac{\left(-\frac{\varphi}{\delta} - \mu\right) - z}{T_z}, \quad (13)$$

$$\sigma = z. \quad (14)$$

For the transformation (11)-(14) to be nondegenerate, it is necessary and sufficient that the determinant:

$$\det \begin{bmatrix} -\frac{\theta}{T_\varphi} & \frac{1-\theta}{T_\varphi} & 0 & 0 \\ 0 & -\frac{1}{T_\xi} & -\frac{1}{T_\xi} & 0 \\ -\frac{1}{T_z \delta} & 0 & -\frac{1}{T_z} & -\frac{1}{T_z} \\ 0 & 0 & 0 & 1 \end{bmatrix} \neq 0 \quad (15)$$

Since the matrix itself is nondegenerate, and in the future, we will assume that the system satisfies it. Consider the resulting indirect regulation frame, since the nature of the feedback signal effect on the regulation object is indirect-through derivatives.

Lyapunov functions for nonlinear stationary indirect control systems.

Now let the control object be an indirect control system

$$\dot{y} = Ay + b\varphi(\sigma)$$
$$\dot{\sigma} = c^* y, \quad (16)$$

where y is an n-dimensional vector, A is a constant square matrix of the n-th order, b and c are n-dimensional constant vectors, and $\varphi(\sigma)$ is a non-linearity characteristic. For now, we will explicitly assume that the matrix A is nondegenerate.

Non-linearity characteristic. The non-linearity of the differential equations frame is determined by the regulation device non-linearity characteristics $\varphi(\sigma)$. Here σ describes the so-called feedback signal, and the non-linearity of the characteristic is explained by the nature of the servo or control device [9]. For mathematical research, only the type of function matters $\varphi(\sigma)$. Consider frames that $\varphi(\sigma)$ belong to the class of acceptable characteristics that satisfy the following conditions:

1.The function $\varphi(\sigma)$ is defined and uninterrupted for all σ values.

2. $\varphi(0)=0$ and $\sigma\varphi(\sigma)>0$ under any $\sigma \neq 0$, i.e., $\varphi(\sigma)$ has the same sign as σ .

3. Integrals $\int\limits_0^{\infty} \varphi(\sigma)d\sigma \;\; u \;\; \int\limits_0^{-\infty} \varphi(\sigma)d\sigma$ diverge.

The first of these conditions assumes that the characteristic has no jumps, the second condition means that the characteristic graph lies only in the first and third quadrants. The third condition shows that the areas under the right and left sections of the curve are infinitely large [10].

Since any investigated frame's critical point corresponds to its solution, the absolute stability of the system requires that the origin (y = 0, σ = 0) be the only critical point of the system (c).

Moving now directly to the solution of the problem of absolute stability, first of all, we find out what the matrix A itself should be. [12]. The system (c) should be asymptotically stable even in the case when the permissible characteristic has the form $\varphi = \mu\,\sigma$, $\mu > 0$. Therefore, the matrix

$$\begin{bmatrix} A & b\mu \\ c' & 0 \end{bmatrix}$$

such a linear frame should not have characteristic numbers with positive real parts. However, for small μs, one of these numbers is small, and the others differ very little from the characteristic numbers of the matrix A. Therefore, the matrix A should not have characteristic numbers with positive real parts. [13]. Then *the first necessary condition for absolute stability is the requirement for the stability of the matrix A*.

Since the presence of purely imaginary characteristic numbers in the matrix A leads to many complications, *let us assume for the time being that the matrix A is stable.*

The only known method for investigating the absolute stability of the system (16) is based on Lyapunov's theorem on asymptotic stability and the addition of Barbashin and Krasovsky. Following Lurie and Postnikov, we will look for the Lyapunov function in the form

$$V = \frac{1}{2} y^* H y + \rho \int_0^{\sigma} \varphi(\tau) d\tau \quad (17)$$

and calculate its time derivative along the trajectory of the (16) system

$$\dot{V} = y^* H(Ay + b\varphi) + \rho\varphi c^* y =$$
$$= \frac{1}{2} y^* (HA + A^* H) y + y^* (Hb + c\rho)\varphi \quad (18)$$

From here we find

$$-\dot{V} = y'Cy$$

where

$$Hb + c\rho = 0,$$
$$A'H + HA = -C . \quad (19)$$

The last equation is an important Lyapunov relation. In accordance with Lyapunov's theorem, it is necessary, first of all, that the function V(y, σ) be positive definite for any values of y and σ. To do this, it needs H>0. If this condition is satisfied, then V>0 for y and σ, and V > 0 for y = 0, but σ≠0, since Φ> 0 (by virtue of condition 2). The latter statement holds for any valid characteristics φ (σ). Finally, since H > 0, the expression $|y'Hy| \to \infty$ with unlimited increase |y|, Φ(σ)→ ∞ with increasing $|y|$ +| σ |. Thus, in order to satisfy the first condition imposed on the function V(y, σ) in the absolute stability theorems, it is sufficient to put H > 0 [14].

Now it remains only to provide positive certainty $-\dot{V}$ under all conditions*y*. In this case, it is very successful that $-\dot{V}$ depends on σ only indirectly through φ (σ) and is a quadratic form with respect to y and φ. Therefore, it is sufficient to require positive definiteness as a quadratic form with regardingto y. According to Sylvester's result, the necessary and sufficient condition for the positive definiteness of this quadratic form is that all major minors of the matrix C>0 are positive.

But since C>0 implies H> 0, the only conditions are $(Hb + c\rho) = 0$.

Indeed, from the conditions C>0 and Hb+$c\rho$=0, follows the absolute stability of the system, and hence the fact that the origin is the only critical point of the system [15-17].

When studying the frame for absolute stability, we will, as a rule, look for some Lyapunov function V (y, o), positively defined with regarding to (y, σ) and such that along the trajectory of the system under study there $-\dot{V}$ is a positive definite quadratic form with regarding to (y, φ) [18-20].

Lyapunov functions for studying the stability of nonlinear control systems of power turbines

Consider the equations of a nonlinear regulation system for power turbines:

$$\dot{y_1} = -a_1 y_1 + a_2 y_2 \tag{20}$$

$$\dot{y_2} = -a_3 y_2 + a_4 \varphi(\sigma) \tag{21}$$

$$\dot{y_3} = -a_5 y_1 - a_6 y_3 - a_7 \varphi(\sigma) \tag{22}$$

$$\dot{\sigma} = y_3 \ , \tag{23}$$

$\varphi(0) = 0$ and $\sigma\varphi(\sigma) > 0$ for any $\sigma \neq 0$, i.e. $\varphi(\sigma)$ has the same sign as σ .

Using the Lyapunov function in the form

$$V = \frac{1}{2} y^* H y + \rho \int_0^{\sigma} \varphi(\tau) d\tau \tag{24}$$

and its time derivative along the trajectory of the system (16)

$$\dot{V} = \frac{1}{2} y^* (HA + A^* H) y + y^* (Hb + c\rho)\varphi \tag{25}$$

Let's check the fulfillment of this condition:

$$Hb + c\rho = 0,$$
$$A'H + HA < 0.$$

Fromequations (20)-(23) weobtain:

$$HA = \begin{vmatrix} h_1 & h_2 & h_3 \\ h_2 & h_4 & h_5 \\ h_3 & h_5 & h_6 \end{vmatrix} \begin{vmatrix} -a_1 & a_2 & 0 \\ 0 & -a_3 & 0 \\ -a_5 & 0 & -a_6 \end{vmatrix} = \begin{vmatrix} -h_1 a_1 - h_3 a_5 & h_1 a_2 - h_2 a_3 & -h_3 a_6 \\ -h_2 a_1 - h_5 a_5 & h_2 a_2 - h_4 a_3 & -h_5 a_6 \\ -h_3 a_1 - h_6 a_5 & h_3 a_2 - h_5 a_3 & -h_6 a_6 \end{vmatrix}$$

$$(HA + A^* H) =$$

$$= \begin{vmatrix} -2h_1 a_1 - 2h_3 a_5 & h_1 a_2 - h_2 a_3 - h_2 a_1 - h_5 a_5 & -h_3 a_6 - h_3 a_1 - h_6 a_5 \\ h_1 a_2 - h_2 a_3 - h_2 a_1 - h_5 a_5 & 2h_2 a_2 - 2h_4 a_3 & -h_5 a_6 + h_3 a_2 - h_5 a_3 \\ -h_3 a_6 - h_3 a_1 - h_6 a_5 & -h_5 a_6 + h_3 a_2 - h_5 a_3 & -2h_6 a_6 \end{vmatrix}$$

Next from the condition $(Hb + c\rho) = 0$ receive:

$$(Hb + C\rho) = (H) \begin{pmatrix} 0 \\ a_4 \\ -a_7 \end{pmatrix} + $$
$$+ \begin{pmatrix} 0 \\ 0 \\ \rho \end{pmatrix} = \begin{pmatrix} h_2 a_4 - h_3 a_7 = 0 \\ h_4 a_4 - h_5 a_7 = 0 \\ h_5 a_4 - h_6 a_7 + \rho = 0 \end{pmatrix}$$

From the higher-level solutions, we derive the elements of the matrix H:

$$h_2 = \frac{h_3 a_7}{a_4}, h_5 = \frac{-\rho + h_6 a_7}{a_4},$$

$$h_1 = \frac{1}{a_2} [(a_3 + a_1) h_3 \frac{a_7}{a_4} + a_5 \frac{\rho + h_6 a_7}{a_4}],$$
$$h_6 = -\frac{1}{a_5} [(a_6 + a_1) h_3]$$
$$h_3 = \frac{1}{a_2} [(a_6 + a_3) h_5] = \frac{(a_6 + a_3)(\rho + h_6 a_7)}{a_2 a_4} =$$
$$= \frac{(a_6 + a_3)\rho}{a_2 a_4} + \frac{(a_6 + a_3) a_7}{a_2 a_4} (-\frac{(a_6 + a_1)}{a_5} h_3)$$

Let $\rho = 1$, then after the transformation we get the matrix:

$$H = \begin{vmatrix} h_1 & h_3 \frac{a_7}{a_4} & h_3 \\ h_3 \frac{a_7}{a_4} & a_7 \frac{(-1 + h_6 a_7)}{a_4{}^2} & \frac{(-1 + h_6 a_7)}{a_4} \\ h_3 & \frac{(-1 + h_6 a_7)}{a_4} & h_6 \end{vmatrix}.$$

Let's check the condition H > 0 of the matrix. Let's assume $h_1 > 0$, the following inequalities are valid from the above equations:

$$h_1 \frac{(a_7 + h_6 a_7^2)}{{a_4}^2} > {h_3}^2 \left(\frac{a_7}{a_4}\right)^2$$

$$h_1 h_6 > {h_3}^2$$

$$\frac{-(a_7 + h_6 a_7^2)}{{a_4}^2} h_6 > \left(\frac{-1 + h_6 a_7}{a_4}\right)^2 => h_6 a_7 > -1 + h_6 a_7. \; h_1 = 2, h_3 = 1, h_6 = \frac{a_7 + 1}{a_7}$$

$$H = \begin{vmatrix} 2 & \frac{a_7}{a_4} & 1 \\ \frac{a_7}{a_4} & \left(\frac{a_7}{a_4}\right)^2 & \frac{a_7}{a_4} \\ 1 & \frac{a_7}{a_4} & \frac{a_7 + 1}{a_7} \end{vmatrix} > 0$$

Substituting the values of the model parameters: $T_z = 0.5, T_\mu = 0.2c, T_\varphi = 12c, T_\xi = 0.1,$ $\theta = 0.05, \Delta z_0 = 0.3, \delta = 0.04$, calculate:

$$H = \begin{vmatrix} 2 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1{,}01 \end{vmatrix} > 0, \Delta_1 > 0, \Delta_2 = 4 > 0, \Delta_3 = 0{,}04 > 0,$$ which I should have proved.

Indeed, the stability conditions of the system are satisfied, and hence the fact that the origin is the only critical point of the system (20)-(23) [20-22].

Numerical calculations. We will write the mathematical model of the regulation object in the form of a differential equations system:

$$\dot{y_1} = -\frac{\theta}{T_\varphi} y_1 + \frac{(1-\theta)}{T_\varphi} y_2 \tag{26}$$

$$\dot{y_2} = -\frac{1}{T_\xi} y_2 + \frac{1}{T_\xi T_\mu} \varphi(\sigma) \tag{27}$$

$$\dot{y_3} = -\frac{1}{T_z \delta} y_1 - \frac{1}{T_z} y_3 - \frac{1}{T_z T_\mu} \varphi(\sigma) \tag{28}$$

$$\dot{\sigma} = y_3 \tag{29}$$

where indicated: $y_1 = (-\theta\varphi + (1-\theta)(\xi - \lambda))/T_\varphi$, $y_2 = (-\xi + \mu)/T_\xi$, $y_3 = \frac{\left(-\frac{\varphi}{\delta} - \mu\right) - z}{T_z}$,

$$\varphi(\sigma) = \begin{cases} 0.5\sigma + 0.25 \frac{\sigma^2}{\Delta z_0} signz, by|\sigma| \le \Delta z_0, \\ \sigma - 0.25 \Delta z_0 signz, by|\sigma| > \Delta z_0, \end{cases} \tag{30}$$

A solution was found for the parameters values $T_z = 0.05, T_\mu = 0.2c, T_\varphi = 12c, T_\xi = 0.1, \theta = 0.05, \Delta z_0 = 0.3, \delta = 0.04$, plotting in the Matlab software product under initial conditions: $y_1(0) = 0.3, y_2(0) = 0.15, y_3(0) = 0.1, y_4(0) = 0.1$.

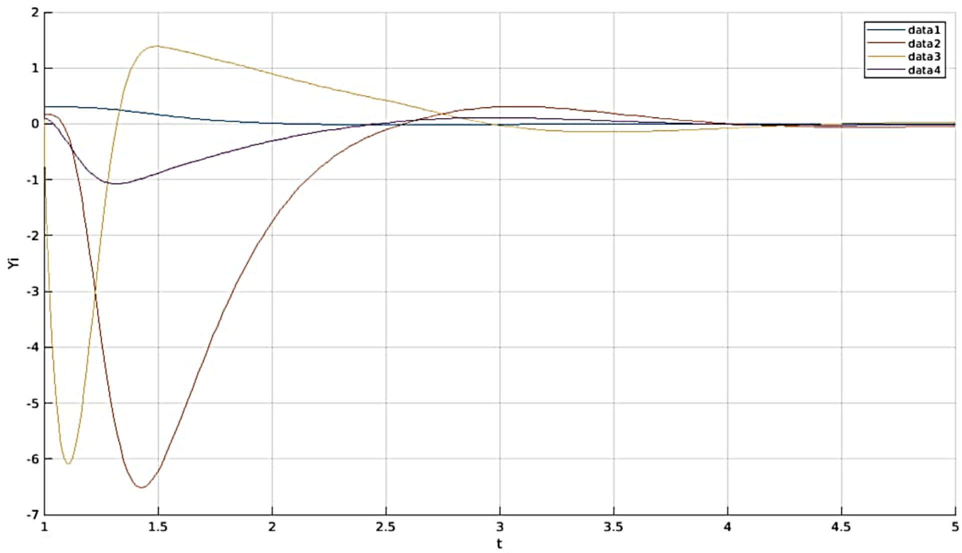

Figure 2 – Changing state variables in the control system

Figure 2 showsthe processes of changing the speed of the rotor data1, changing the steam flow data2, the cut-off spool data3 and the servomotor of the turbine drive data4. All processes proceed smoothly without significant fluctuations.

Conclusions

The dynamics of the& turbine regulation system is described by a nonlinear& system with &four differential equations. The mathematical model of the turbine in the place of states is based on the differential equations of the tracking device, the steam path &and the equation& of the rotor displacement. Changing state variables of the steam turbine regulation system are constructed. Because all conclusions, independently of the choice of the optimal property, converge to the origin, the system is unconditionally stable, in fact, which justifies the adequacy of the model. Numerical calculations are performed for a system of differential equations &and graphs are built in the Matlab_software&product.

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