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Exact solution of the fiels equation of the generalized cosmological F(R,T) model

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We considered cosmological model within the framework of the generalized F(R,T) gravity in flat FLRW metric. Earlier, an exponential solution was shown for the generalized case, which describes the inflationary stage of the evolution of the Universe. In this paper, it is shown that for the particular case of the Starobinsky model, the generalized model has, among other things, a power-law solution that describes the dust-like stage and the stage of radiation dominance. It should be noted that this solution was obtained by the analytical method as a solution to the Euler-Lagrange equation for this model, taking into account the functions u, v – describing the relationship between the curvature scalar and the torsion scalar, and depending not only on the first, but also on the second time derivative of the scale factor. This solution is of interest, since it shows not only that this model has a solution, but also that this solution can describe the modern observable Universe for more complex forms u, v than described earlier. This solution allows other related cosmological parameters to be obtained accordingly. The paper presents the parameter ω for the equation of state, which describes the stage of the evolution of the Universe.

Key words: generalized cosmological model, scale factor, Starobinsky model.
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Introduction

Observation of the distribution of matter in the Universe earlier led not only to the idea of the Big Bang theory, but also to Starobinsky's model describing the early accelerated expansion of the Universe, as a consequence, leading to the observed homogeneity of the Universe [6]. Later, these assumptions were confirmed by many independent studies [7-10], but the reason leading to this model has not yet been determined. We have previously assumed that one of the reasons may be the symmetry of the generalized model [11]. However, here you can face the problem that generalized cosmological models become too mathematically complicated, and cannot be solved analytically.

So, already taking into account the presence of the functions u, v – describing the connection between the curvature scalar and the torsion scalar, which should help to give in the limit, respectively, the general theory of relativity or teleparallel – gravity [12-15] is already so difficult to compute that researchers often try to describe the solution without taking into account these functions [16]. As a first approximation, it is possible to take into account these components as functions of the scale factor and its first time derivative. But it is important to take into account that in fact they have more complex dependencies $u = u(\Gamma^{\rho}_{\mu\nu}; x_i; g_{ij}, \dot{g}_{ij}, \ddot{g}_{ij}, \dots; f_i)$, $v = v(\Gamma^{\rho}_{\mu\nu}; z_i; g_{ij}, \dot{g}_{ij}, \ddot{g}_{ij}, \dots; g_i)$. Here, it is important for us to understand whether the generalized cosmological model that we are considering satisfactorily describes the observed Universe, also satisfies functions u, v depending not only on the first, but also on the second time derivative of the scale factor. This work is devoted to this study.

Model

In this work we use Friedmann–Lemaître–Robertson–Walker metric [1-5]

$$ds^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2), \qquad (1)$$

where $a(t)$ – scale factor. Here $F(R,T)$ is denoted as F.

The action for the model has the following form:

$$S = 2\pi^2 \int dt a^3 (F - \lambda_1 \left[R - u + 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\right] - -\lambda_2 \left[T - v + 6\left(\frac{\dot{a}^2}{a^2}\right)\right]. \quad (2)$$

Here – Ricci scalar, T – torsion scalar and u, v- functions, that depend on $a, \dot{a}, \ddot{a}$. Here and further dots represent derivation with respect to the cosmic time.

Values of λ_1 and λ_2 were obtained by varying the action with respect to R and T:

$$\lambda_1 = F_R,\ \lambda_2 = F_T. \quad (3)$$

Indices of F express the derivatives over specified variable.

Now we can write the Lagrangian in its points-like form using the Lagrange multipliers λ_1, λ_2, obtained previously:

$$L = a^3[F - (R-u)F_R - (T-v)F_T] + \\ +6a\dot{a}^2[F_R - F_T] + 6a^2\dot{a}[\dot{R}F_{RR} + \dot{T}F_{RT}], \quad (4)$$

Solution

Deriving the Euler- Lagrange equation for the scale factor a. Since in this work we take into account second derivative of a, then the formula will have to take the following form:

$$\frac{\partial L}{\partial a} - \frac{d}{dt}\frac{\partial L}{\partial \dot{a}} + \frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{a}} = 0, \quad (5)$$

Putting Lagrangian (4) into equation (5), we obtain the equation of motion:

$$F + AF_R + BF_T + \left(4H - u_{\dot{a}}\frac{a}{3} + 2\dot{a}u_{\ddot{a}} + \frac{2}{3}a(u_{\ddot{a}a}\dot{a} + u_{\ddot{a}\dot{a}}\ddot{a} + u_{\ddot{a}\ddot{a}}\dddot{a})\right)\dot{F}_R + \\ + \left(-v_{\dot{a}}\frac{a}{3} - 4H + 2\dot{a}v_{\ddot{a}} + \frac{2}{3}a(v_{\ddot{a}a}\dot{a} + v_{\ddot{a}\dot{a}}\ddot{a} + v_{\ddot{a}\ddot{a}}\dddot{a})\right)\dot{F}_T - \\ - \left(2 + u_{\ddot{a}}\frac{a}{3}\right)\ddot{F}_R + v_{\ddot{a}}\frac{a}{3}\ddot{F}_T = 0, \quad (6)$$

here $H = \frac{\dot{a}}{a}$- Hubble parameter, and by A and B we mean the following expressions:

$$A = -R + u + \frac{a}{3}u_a - u_{\dot{a}}\dot{a} - \frac{a}{3}(u_{\dot{a}a}\dot{a} + u_{\dot{a}\dot{a}}\ddot{a} + u_{\dot{a}\ddot{a}}\dddot{a}) + u_{\ddot{a}}\left(2\frac{\dot{a}^2}{a} + \ddot{a}\right) + \\ +2\dot{a}(u_{\ddot{a}a}\dot{a} + u_{\ddot{a}\dot{a}}\ddot{a} + u_{\ddot{a}\ddot{a}}\dddot{a}) + \\ + \frac{a}{3}[\dot{a}(u_{\ddot{a}aa}\dot{a} + u_{\ddot{a}a\dot{a}}\ddot{a} + u_{\ddot{a}a\ddot{a}}\dddot{a}) + u_{\ddot{a}a}\ddot{a} + \ddot{a}(u_{\ddot{a}\dot{a}a}\dot{a} + u_{\ddot{a}\dot{a}\dot{a}}\ddot{a} + u_{\ddot{a}\dot{a}\ddot{a}}\dddot{a}) + \\ +u_{\ddot{a}\dot{a}}\dddot{a} + \dddot{a}(u_{\ddot{a}\ddot{a}a}\dot{a} + u_{\ddot{a}\ddot{a}\dot{a}}\ddot{a} + u_{\ddot{a}\ddot{a}\ddot{a}}\dddot{a}) + u_{\ddot{a}\ddot{a}}\ddddot{a}] - \\ -4\dot{H} - 6H^2, \quad (7)$$

$$B = -T + v + \frac{a}{3}v_a - v_{\dot{a}}\dot{a} - \frac{a}{3}(v_{\dot{a}a}\dot{a} + v_{\dot{a}\dot{a}}\ddot{a} + v_{\dot{a}\ddot{a}}\dddot{a}) + \\ +v_{\ddot{a}}\left(2\frac{\dot{a}^2}{a} + \ddot{a}\right) + 2\dot{a}(v_{\ddot{a}a}\dot{a} + v_{\ddot{a}\dot{a}}\ddot{a} + v_{\ddot{a}\ddot{a}}\dddot{a}) + \\ + \frac{a}{3}[\dot{a}(v_{\ddot{a}aa}\dot{a} + v_{\ddot{a}a\dot{a}}\ddot{a} + v_{\ddot{a}a\ddot{a}}\dddot{a}) + v_{\ddot{a}a}\ddot{a} + \ddot{a}(v_{\ddot{a}\dot{a}a}\dot{a} + v_{\ddot{a}\dot{a}\dot{a}}\ddot{a} + v_{\ddot{a}\dot{a}\ddot{a}}\dddot{a}) + v_{\ddot{a}\dot{a}}\dddot{a} + \\ +\dddot{a}(v_{\ddot{a}\ddot{a}a}\dot{a} + v_{\ddot{a}\ddot{a}\dot{a}}\ddot{a} + v_{\ddot{a}\ddot{a}\ddot{a}}\dddot{a}) + v_{\ddot{a}\ddot{a}}\ddddot{a}] + \\ +4\dot{H} + 6H^2. \quad (8)$$

This equation cannot be solved analytically. However, we can take F in the form of a generalized Starobinsky model, for which $F = R + R^2 + T + T^2$, found by us in this form earlier [16].

Suppose that the functions $u = n\ddot{a}, v = m\ddot{a}$ linearly depend on the second derivative with respect to a. We get the equation of motion for a new model:

$$\begin{aligned} &12\dot{a}[\ddot{a}\dot{a}(na^2 - 36a) - 6\dot{a}^3 + a^2\dot{a}\ddot{a}(2n - m) + \ddot{a}(na\dot{a}^2 - 6a^2)] = \\ &= a^2[-a^2(n\ddot{a} - 6\ddot{a})^2 + 24n\dot{a}^2\ddot{a}(na - 6) - 180\dot{a}^4 - m^2\ddot{a}^2a^4 + \\ &+12a^2\dot{a}^2\ddot{a} + \ddot{a}(n + m - 36\dot{a}^2 + 2\ddot{a}a(na - 6 + ma))]. \end{aligned} \quad (9)$$

This equation is difficult to solve explicitly. But we have shown that its solution is satisfied by the substitution of a scale factor with a power-law dependence on time in the following form [17]

$$a = a_0 t^s. \quad (10)$$

Results

Hubble parameter here is $= \frac{s}{t}$. The equation of state is as follows:

$$P = \omega\rho c^2. \quad (11)$$

The parameter ω of the equation of state has the following form:

$$\omega = -1 + \frac{2}{3s}. \quad (12)$$

Deceleration parameter:

$$q = -\left(1 + \frac{\dot{H}}{H^2}\right) = \frac{1}{s} - 1, \quad (13)$$

The slow roll parameter:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{s}. \quad (15)$$

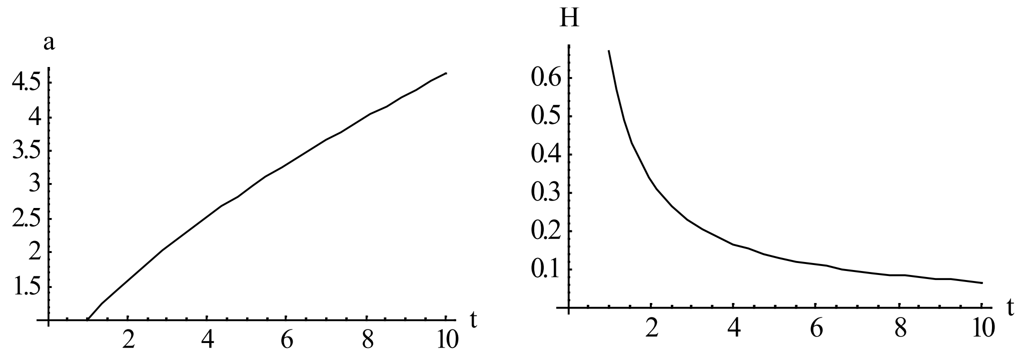

Figure 1 – Form of scale factor a (left) and Hubble parameter H (right).

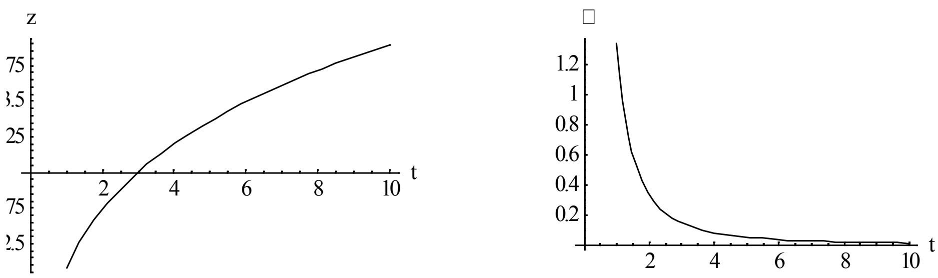

Figure 2 – Time dependence of the redshift z for a power-law cosmological model (left) and dependence of the density ρ (right).

Conclusions

In this work, we considered a generalized model F (R, T) of gravity, defined in the FLRW metric, with functions u and v depending not only on the scale factor and its first derivative, but also on the second one. It was found that the solution for the scale factor of this model could be obtained in a power-law form $a = a_0 t^s$, after taking F in the form of a generalized Starobinsky model. This shows not only that this model not only has a solution for the simplest cases, but also well describes the observable Universe for more complex solutions. It is obvious that this method can be considered on other generalizations [18], like k-essence [19], quintessence field models [20-21] and other dark energy scenarios. This is important, since generalized model ultimately gives the effect of the initial inflation of the Universe. The result brings new opportunities for future research. Further, we can consider studying and observing other models or we can deal with obtaining the parameters of the Universe for this one. In this work, we have derived only the form of the parameter ω of the equation of state for our case. The related cosmological parameters are of physical interest.

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