

Standard model and predictions for the Higgs boson

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(Received 10 August 2022; received in revised form 22 September; accepted 5 November 2022)

This article describes the electroweak interactions of the Standard Model (SM) with Brout-Englert-Higgs (BEH) mechanism, the appearance and properties of the Higgs boson. Brout-Engler-Higgs mechanism plays an important role in the Standard Model, which introduce a scalar Higgs field with a non-zero vacuum expectation results from spontaneous symmetry breaking. So, due to the interaction with this field, elementary particles become massive. Here arise electrically neutral quanta associated with the Higgs field, so called Higgs boson, in the same way that there is a quantum associated with the electromagnetic field, i.e., photons. The strong part of the Standard Model, Quantum Chromodynamics, quark mixing Cabibbo–Kobayashi–Maskawa matrix, physics of the neutrino and many other issues were not mentioned in the article. Brout-Englert-Higgs mechanism and associated Higgs boson are important parts of the Standard Model of elementary particles. In this paper we present the full Lagrangian of the Standard Model and a set of theoretical predictions about the Higgs boson before its detection, as well as the current status of the Higgs boson problem.

Keywords: Standard Model, High Energy, Large Hadron Collider, Higgs boson, Higgs mechanism, Spontaneous symmetry breaking.

PACS number: 14.65.–q

1 Introduction

In 2013, a year after the discovery of the Higgs boson [1, 2] at the Large Hadron Collider, the Nobel Prize was awarded. This was an important stage in elementary particle physics. This was an experimentally confirmation of the Brout – Engler – Higgs mechanism of spontaneous symmetry breaking, which was theoretically predicted about half century ago [3, 4]. The Standard Model (SM) of fundamental interactions between elementary particles was finally completed and acquired the status of a standard theory.

SM describes strong, weak and electromagnetic interactions between elementary particles on the basis of gauge theory with $SU(3) \times SU(2) \times U(1)$ symmetry and by using quantum field theory. According to the SM, there are six quarks and six leptons, which are divided into three generations. They interact with each other by three types of interactions, such as strong interaction, weak interaction and electromagnetic interaction. The

forces between interacting particles are described by exchange of corresponding quantum fields such as, gluons, W- and Z-bosons and a photon.

Strong interactions are responsible for the attraction of protons and neutrons and provide the stability of the nuclei themselves. These interactions are short-range with a characteristic radius of action of the order of $10^{-12} - 10^{-13}$ cm, which is the size of the nucleus. Electromagnetic interaction is responsible for the stability of atoms and molecules due to the interaction of positively charged atomic nuclei and negatively charged electrons. The radius of these forces can be very large. We cannot feel it directly, because atoms and molecules as a whole are electrically neutral. Weak interactions are responsible for the decay of a free neutron and the instability of a number of atomic nuclei. This interaction provides nuclear cycles due to which occurs the release of energy on the Sun.

The huge world surrounding us is built only from leptons and quarks of the first generation. Quarks and leptons of other two generations manifest themselves

in high-energy cosmic rays or in laboratory conditions at accelerators, colliders. Six quarks and six leptons (and their corresponding antiparticles), as well as vector bosons (photon, gluon, W and Z boson) and the recently discovered Higgs boson construct a set of fundamental particles of the SM.

2 Standard Model

The SM of particle physics provides a successful description of all current experimental data and shows a unified picture of the elementary particles, and the interactions between them. In the SM electromagnetic interaction between quarks and leptons are carried out by the exchange of photons, weak interaction between fundamental particles are carried out by the exchange of W and Z bosons, and the strong interaction between quarks are carried out by the exchange of gluons. The standard theory of particle physics with local gauge invariance defined by the $SU_c(3) \times SU_L(2) \times U(1)_Y$ symmetry groups has been experimentally confirmed with high accuracy in lots of experiments. Withal the problem of the origin of all particle masses remained unclear. The exact gauge symmetry $SU_L(2) \times U(1)_Y$ requires the particles be massless. But it is known, for example, that the masses of the W and Z bosons exceed the mass of the proton by almost a hundred times. The elementary particles could be massive only when the gauge symmetry $SU_L(2) \times U(1)_Y$ is broken. From here, scientists have suggested the existence of a self-acting scalar field that fills the entire universe, the so-called Higgs field. Interacting with the vacuum condensate of the Higgs field, the particles become massive. This mechanism of the origin of masses is called “spontaneous” symmetry breaking, all field interactions preserve the gauge symmetry and the Higgs field breaks it. The quanta of the new scalar field, so called Higgs bosons, would be produced in the collision of electron-positron or high-energy proton-antiproton beams.

The SM must satisfy the following requirements:

1. The reproducibility of electromagnetic interactions of leptons and quarks which is invariant under the group of $U(1)_{em}$ transformations.
2. The reproducibility of the axial-vector structure (V-A) of the charged currents of leptons and quarks.
3. The independence of the Lagrangian from the phase of the fields which means the gauge character of the interactions.

4. Renormalizability and unitarity which means that the mass dimension of the operators used in the Lagrangian should be not exceed four.

5. The absence of the chiral anomalies.

6. The possibility of considering all generations of leptons and quarks.

7. The possibility to describe massive fermions and gauge bosons without violation of the local gauge invariance. All fermions (possibly except for one of the neutrinos) and electroweak gauge bosons W, Z are massive particles. To describe the massive fields it was introduced the hypothesis of spontaneous symmetry breaking in which the gauge symmetry of the Lagrangian remains exact while the vacuum of the theory violates this gauge symmetry.

Let take the SM Lagrangian composed of the gauge invariant operators of dimension no higher than 4 as a starting point:

$$L = -\frac{1}{4} W_{\mu\nu}^i (W^{\mu\nu})^i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G_{\mu\nu}^a (G^{\mu\nu})^a + \sum_{f=1,q} \bar{\Psi}_L^f (iD_\mu^L \gamma^\mu) \Psi_L^\dagger + \sum_{f=1,q} \bar{\Psi}_R^f (iD_\mu^R \gamma^\mu) \Psi_R^\dagger. \quad (1)$$

where,

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i + g_2 \varepsilon^{ijk} W_\mu^j W_\nu^k, \\ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu, \\ G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c, \quad (2)$$

$$D_\mu^L = \partial_\mu - ig_2 W_\mu^i \tau^i - ig_1 B_\mu \left(\frac{Y_L^f}{2} \right) - ig_s A_\mu^a t^a, \\ D_\mu^R = \partial_\mu - ig_1 B_\mu \left(\frac{Y_R^f}{2} \right) - ig_s A_\mu^a t^a, \quad (3)$$

here the each indices of i, j, k have the values 1, 2, 3 and the indices a, b, c have the values 1, ..., 8; $Y_{L,R}^f$ is a weak hypercharge for the left and right fields of the quarks and leptons.

Further, we rewrite the Lagrangian (1) in terms of charged vector fields $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ and neutral fields W_μ^3 and B_μ , which are mixtures of other fields, A_μ and Z_μ with some mixing angle θ_W (Weinberg mixing angle):

$$W_\mu^3 = Z_\mu \cos\theta_W + A_\mu \sin\theta_W,$$

$$B_\mu = -Z_\mu \sin\theta_W + A_\mu \cos\theta_W \quad (4)$$

The interaction Lagrangian for the charged currents (CC) is immediately obtained from (1) and has the desired form (V-A):

$$L_{CC}^l = \frac{g_2}{\sqrt{2}} \bar{\nu}_{eL} \gamma_\mu W_\mu^+ e_L + h.c. =$$

$$= \frac{g_2}{2\sqrt{2}} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) W_\mu^+ e + h.c., 0 \quad (5)$$

$$L_{CC}^q = \frac{g_2}{2\sqrt{2}} \bar{u} \gamma_\mu (1 - \gamma_5) W_\mu^+ d +$$

$$+ \frac{g_2}{2\sqrt{2}} \bar{d} \gamma_\mu (1 - \gamma_5) W_\mu^- u \quad (6)$$

For the interaction of neutral currents (NC) according to the requirement 1, it is necessary that one of the fields, let's say A_μ , have the correct electromagnetic interaction like the photon field. This leads to a set of equations for hypercharges that have solutions of the form: $Y_R^e = 2Y_L^e, Y_R^u = \frac{-4}{3}Y_L^e, Y_R^d = \frac{2}{3}Y_L^e$, where the hypercharges are expressed in terms of one independent parameter, for example, the left lepton doublet hypercharge Y_L^e [5]. One of these equations gives the relation between the charges and the mixing angle: $g_2 \sin\theta_W = -g_1 Y_L^e \cos\theta_W$. In this case, the Lagrangian of neutral currents with the fields A_μ and Z_μ takes the following form:

$$L_{NC} = e \sum_f Q_f J_{f\mu}^{em} A^\mu + \frac{e}{4\sin\theta_W \cos\theta_W} \sum_f J_{f\mu}^Z Z^\mu \quad (7)$$

where, $J_{f\mu}^{em} = \bar{f} \gamma_\mu f, Q_\nu = 0, Q_e = -1, Q_u = \frac{2}{3}, Q_d = -\frac{1}{3}, J_{f\mu}^Z = \bar{f} \gamma_\mu (\vartheta_f - a_f \gamma_5) f$. As a result, the Lagrangians (5) and (7), which are obtained from the Lagrangian (1), have a structure that demonstrates the next "good" properties of the theory:

- 1) correct charged currents (V - A);
- 2) correct electromagnetic interactions;

- 3) reduction of chiral anomalies;
- 4) prediction of new neutral currents and new Z_u boson interacting with them.

These currents and the boson are detected experimentally.

However this constructed theory cannot correctly describe the properties of nature. All fields in this theory must be massless. But in nature all charged leptons and quarks, W and Z bosons have nonzero masses. The introduction of the masses into the theory lead to the serious problems. The mass terms for bosons $M_V^2 V_\mu V^{\mu*}$ and the Dirac mass terms for fermions $m_\psi \bar{\Psi} \Psi = m_\psi (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L)$ do not satisfy the main requirement of gauge invariance: the mass term for a vector field is not invariant when $V_\mu \rightarrow V_\mu + \partial_\mu \alpha$, and in the mass term of the fermions the left doublet Ψ_L and the right singlet Ψ_R are transform differently on phase rotation.

3 The Brout-Englert-Higgs mechanism of spontaneous symmetry breaking

The Higgs mechanism is used in the framework of the SM to introduce massive vector and fermion fields without violating the basic principle of gauge invariance in field interactions. Let's add to the expression (1) the Lagrangian of a complex scalar field

$$L_\Phi = D_\mu \Phi^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda^4 (\Phi^\dagger \Phi) \quad (8)$$

which is gauge-invariant for the same group $SU_L(2) \times U_Y(1)$. Here Φ is a complex doublet. Covariant derivative has the following form

$$D_\mu = \partial_\mu - i g_2 W_\mu^i \tau^i - i g_1 \frac{Y_h}{2} B_\mu.$$

Here Y_h is hypercharge of the scalar so called Higgs field. The potential of this field shown in Figure 1 has a minimum for negative μ^2 :

$$|\varphi_0| = \sqrt{\frac{|\mu^2|}{2\lambda}} = \frac{v}{\sqrt{2}} > 0.$$

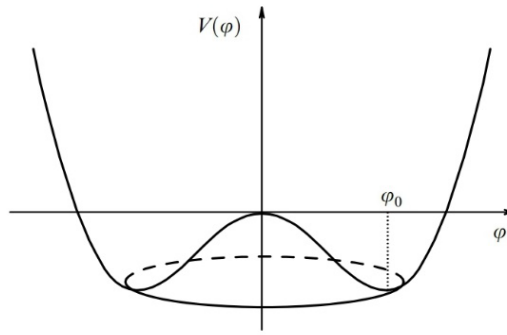


Figure 1 – Potential of the Higgs field

Any particular vacuum solution breaks the symmetry for the phase shift. An arbitrary complex scalar field, which is a doublet of the group $SU_L(2)$, can be parameterized by four real fields:

Since the Lagrangian (8) is invariant under the gauge transformation

$$\Phi(x) \rightarrow \Phi'(x) = \exp(i g_2 \alpha^i \tau^i) \Phi(x),$$

we write as $g_2 \alpha^i(x) = \xi^i(x) / \vartheta$, so that all fields $\xi^i(x)$ with the corresponding gauge field transformation disappear from the Lagrangian and only physical degrees of freedom remain in it. In this so called unitary gauge transformation, after substituting the fields $W_\mu^1, W_\mu^2, W_\mu^3$ and B_μ expressed in terms of the fields W_μ^\pm, A_μ, Z_μ and $\Phi(x) = \begin{pmatrix} 0 \\ \frac{\vartheta + h(x)}{\sqrt{2}} \end{pmatrix}$, the Lagrangian L_ϕ takes the form

$$L = \frac{1}{2} (\partial_\mu h)^2 - \frac{1}{2} (2\lambda\vartheta^2)h^2 - \lambda\vartheta h^3 - \frac{\lambda}{4} h^4 + M_W^2 W_\mu^+ W^{\mu-} \left(1 + \frac{h}{\vartheta}\right)^2 + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \left(1 + \frac{h}{\vartheta}\right)^2. \tag{10}$$

Here we introduce a new field of the new scalar boson (Higgs boson) $h(x)$ with mass $M_h^2 = 2\lambda\vartheta^2$. Also it is included mass terms $M_W = \frac{1}{2} g_2 \vartheta$, $M_Z = \frac{1}{2} (g_2 \cos\theta_W + g_1 Y_h \sin\theta_W) \vartheta$ for the W_μ^\pm and Z_μ boson fields respectively, as well as the interaction term of the W, Z fields with the Higgs boson and the self-interaction terms of the Higgs boson: h^3 and h^4 .

Note that the fields $\xi^i(x)$ in expression (9) are extracted from the Lagrangian by the unitary gauge transformation. However, according to the Goldstone theorem, there must be the existence of $4-1=3$ massless bosons. In the gauge theory, these three bosons become the longitudinal components of the vector fields W^\pm, Z which from massless with two degrees of freedom become massive with three degrees of freedom. This mechanism of mass production is called the Brout-Englert-Higgs mechanism [3,4]. The Brout-Englert-Higgs mechanism makes it possible to give masses not only to gauge bosons, but also to fermions.

Note the one significant problem of the simplest Higgs SM mechanism: it is unstable for quantum loop corrections to the mass of the Higgs boson itself (the problem of the hierarchy). In Figure 2 it is shown the diagrams contributing to the correction to the mass of the Higgs boson

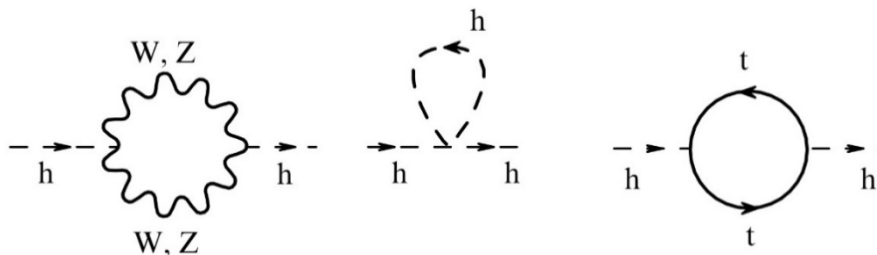


Figure 2 – The diagrams contributing to the mass of the Higgs boson

The correction $\delta M_h^2 = \frac{3G_F}{4\sqrt{2}\pi^2} (2M_W^2 + M_Z^2 + M_h^2 - 4M_t^2)\Lambda^2 \approx -(0.2\Lambda)^2$ corresponding to these diagrams has square dependence on the scale Λ . The scale Λ related to any “new physics”, i.e. any new objects that can contribute to the mass parameter of the Higgs boson. The corrections to the masses of all other SM particles depend on parameter Λ weakly. There is no such symmetry that would forbid such a strong dependence of mass on scale in the case of the scalar Higgs boson. If we require that the corrections to the Higgs boson mass do not exceed the mass itself, $\delta M_h < M_h$, then the limit for Λ would be less than 1 TeV, which is in the contradiction with the data, since none “new physics” has been found on this scale. This difficulty, possibly technical, is called the “problems of small hierarchies.” To solve it, something is needed in addition to SM.

4 The Higgs boson predictions.

What did we know about the Higgs boson before its detection? Let’s present the Lagrangian of the Higgs sector in the SM where the corresponding boson and fermion parts are present:

$$L_h = \frac{1}{2}(\partial^\mu h)(\partial_\mu h) + \frac{M_h^2}{2}h^2 - \frac{M_h^2}{2\vartheta}h^3 - \frac{M_h^2}{8\vartheta^2}h^4 + \left(M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu\right)\left(1 + \frac{h^2}{\vartheta}\right) - \sum_f m_f \bar{f} f \left(1 + \frac{h}{\vartheta}\right) \quad (11)$$

In this formula all coupling constants are also expressed in terms of the Higgs boson mass parameter. The values of the masses of W, Z bosons, leptons and quarks are known from experimental data.

In the Lagrangian (11) all parameters were known from the experiment before the Higgs boson discovery, except the mass of the Higgs boson. This circumstance made it possible to carry out various calculations for different mass regions of the Higgs boson and to obtain several quantitative predictions and experimental limitations.

Assumptions for the decay width and production cross sections for various modes. From the Lagrangian (11) it follows the Feynman rules for the interaction of the Higgs boson with W^\pm and Z bosons

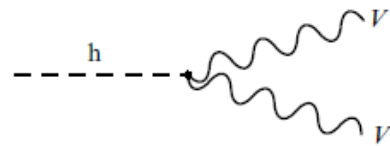
and fermions, as well as the self-interaction h^3 and h^4 vertices of the Higgs boson. We can calculate the main decay widths and production cross-sections for the Higgs boson by using the Feynman rules and taking into account the higher order strong, and in some cases electroweak corrections [6].

Further we introduce the Feynman diagrams for the main Higgs boson decay modes:

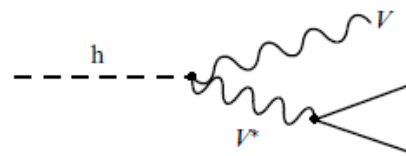
- decays into a fermion and antifermion, $f = \acute{b}, \acute{c}, \tau, \mu$



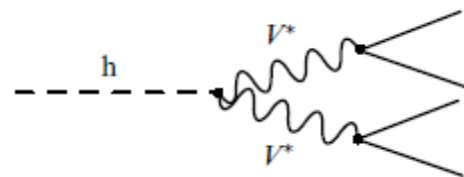
- decays into a real vector bosons W, Z ($M_h > 2M_V$)



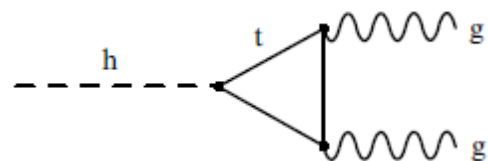
- decays into a real and virtual bosons ($M_V < M_h < 2M_V$)



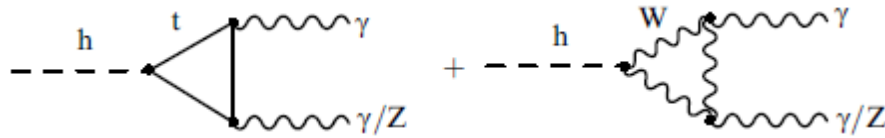
- decays into two virtual bosons ($M_V > M_h$)



- two gluons decays



- decays into two photons, photon and Z-boson



The behaviour of the total width for Higgs boson decays and the partial widths for different modes

depending on Higgs boson mass are shown in Figures 3, 4 [6].

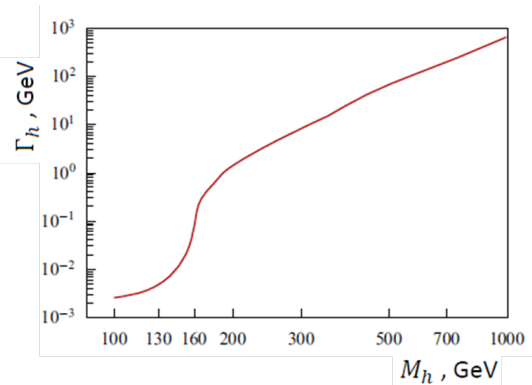


Figure 3 – Dependence of the total Higgs boson decay width Γ_h on its mass M_h [6]

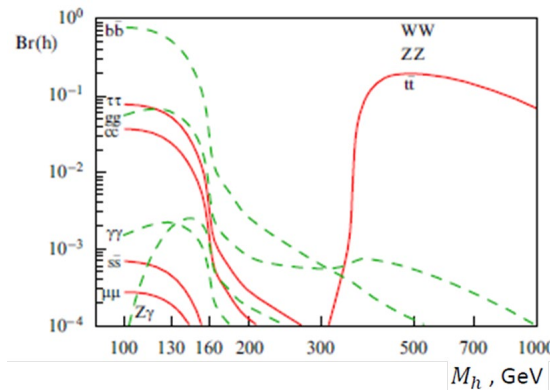


Figure 4 – Partial probabilities of the decay of the Higgs boson through various channels depending on its mass [6]

Limitations from the conditions of unitarity of the SM. The most "dangerous" in terms of energy growth are the amplitudes with participation of the longitudinal components of the W and Z bosons. According to the Brout-Englert-Higgs mechanism

these longitudinal components occur from the corresponding Goldstone bosons. Then the scattering amplitudes of two longitudinal components of the W boson can be found from the goldstone boson scattering diagrams:

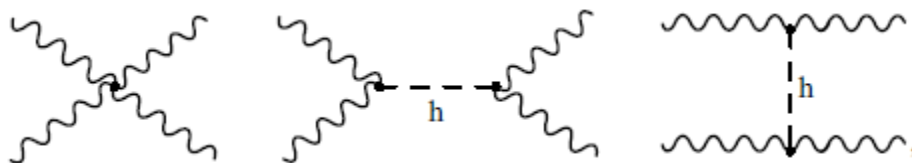


Figure 5 – The goldstone boson scattering diagrams

$$A(W^+W^- \rightarrow W^+W^-) = - \left[2 \frac{M_h^2}{\vartheta^2} + \left(\frac{M_h^2}{\vartheta} \right)^2 \frac{1}{s-M_h^2} + \left(\frac{M_h^2}{\vartheta} \right)^2 \frac{1}{t-M_h^2} \right]. \quad (12)$$

Hence for the amplitude of the zero partial wave we get

$$a_0 = \frac{1}{16\pi s} \int_s^0 dt |A| = - \frac{M_h^2}{16\pi\vartheta^2} \left[2 + \frac{M_h^2}{s-M_h^2} - \frac{M_h^2}{s} \log \left(1 + \frac{s}{M_h^2} \right) \right]. \quad (13)$$

Then the condition $|Re a_1| < 1/2$ will lead to two possible consequences:

$$a_0 \rightarrow - \frac{M_h^2}{8\pi\vartheta^2}, M_h \leq 870 GeV$$

$$a_0 \rightarrow - \frac{s}{32\pi\vartheta^2}, \sqrt{s} \leq 1.7 GeV \quad (14)$$

Therefore, to produce unitarity there should be a Higgs boson with a mass of less than 870 GeV (710 GeV, taking into account all $V_L V_L$ scattering channels), otherwise if $\sqrt{s} \leq 1.7 TeV$ (1.2 TeV) the SM does not work and something outside the CM must manifest itself to ensure unitarity.

Limitations from the condition of self-consistency of the SM.

Limitations arise from consideration of the equation of the renormalization group for the evolution of the self-interaction constant of the Higgs boson

$$\frac{d\lambda}{d\ln Q^2} \simeq \frac{1}{16\pi^2} \left\{ 12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}[2g_2^4 + (g_2^2 + g_1^2)^2] \right\}$$

where $\lambda_t = m_t/v$ is Yukawa coupling constant for top quark and the relation between λ and M_h is $M_h^2 = 2\lambda v^2$ [7].

Limitations from direct searches on colliders. Direct limitations for the Higgs boson mass M_h were obtained on colliders Tevatron and LEP, before the start of the LHC. The LEP studied the process with the Higgs boson emission (the process is called Higgs strahlung):

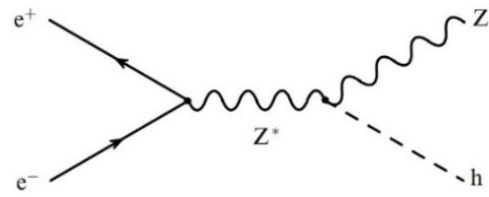


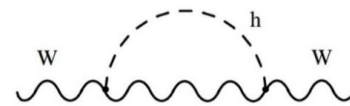
Figure 6 – The Higgs emission process

and obtained the value $M_h > 114.4 GeV$ for lower limit at 95% Confidence Level (CL).

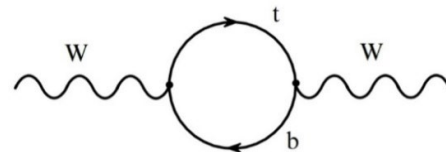
At the Tevatron the Higgs boson mass was excluded in the mass interval $M_h = 160 \div 170 GeV$ when studying the process of gluon fusion with the Higgs boson decay into two W-bosons.

Limitations from comparing the loop corrections of the mass with precision measurements. At the Tevatron and LEP colliders a very important limitations are associated with precision measurements of the W boson mass and the top quark mass: $M_W = 80385 \pm 15 MeV, M_t = 173.18 \pm 0.56(stat) \pm 0.75(syst) GeV$.

It should be noted that there are loop corrections to the boson masses, which depend (logarithmically) on the Higgs boson mass, as shown in the following diagram



and (quadratically) on the top quark mass, as shown in the next diagram



Comparison of these mass shifts and measurement accuracies allows us to establish limitations from above: $M_h < 155 GeV, 95\% CL$.

It should be noted that the comparison of this precision measurements from the Tevatron, LEP and SLC colliders with the results of the SM (taking into account the quantum corrections to the leading approximation) allowed us to demonstrate how well the data were described. These comparisons are shown in Figure 7.

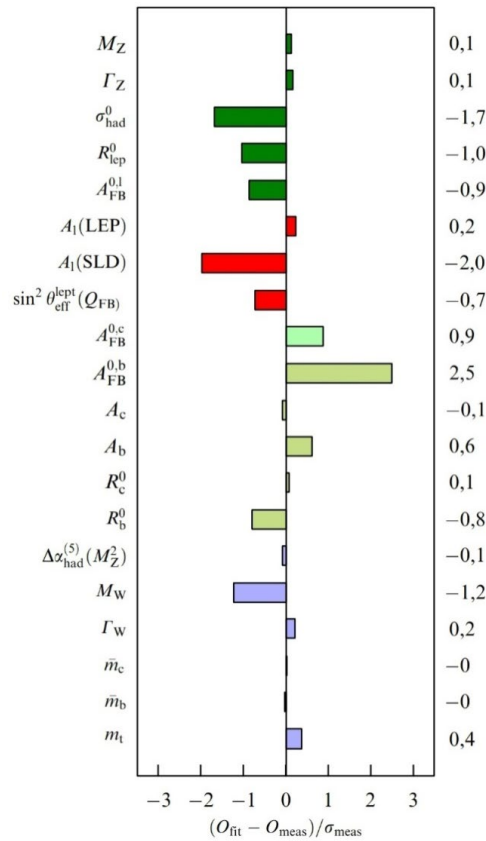


Figure 7 – Global approximation of the precision experimental data for the measurement of some quantities which indicated in the left column of the figure, with the results of calculations of these quantities in the SM taking into account the contribution of loop corrections [8]

5 Higgs boson results

The discovery of the Higgs boson in 2012 completed the SM of elementary particle physics. After that, in the ten years following the discovery, the great progress has been made in painting a clearer portrait of the Higgs boson. The decay rates and production of the Higgs boson were measured using the dataset collected by the CMS and ATLAS experiments during Run 2 of the LHC from 2015 to 2018 [9,10]. The obtained results were in excellent agreement with the Standard Model predictions. All measurements made so far are found to be consistent with the expectations of the SM. In particular, the overall signal-strength parameter has been measured to be $\mu = 1.002 \pm 0.057$ [9]. It has been shown that the Higgs boson directly couples to bottom quarks, tau leptons and muons, which had not been observed at the time of the discovery, and also proven that it is indeed a scalar particle. The CMS experiment is approaching the sensitivity that necessary to probe

Higgs boson couplings to charm quarks [11]. The observed (expected) 95% CL value for κ_c is found to be $1.1 < \kappa_c < 5.5$ ($\kappa_c < 3.40$), the most stringent result so far.

In different scenarios, the couplings of the Higgs boson to the three heaviest fermions, the top quark, the b quark and the τ lepton, were measured with uncertainties ranging from about 7% to 12% and the couplings to the weak bosons (Z and W) were measured with uncertainties of about 5% [10]. A comprehensive study of Higgs boson production kinematics was performed and the results were also in good agreement with the Standard Model predictions.

With the Run 2 data, CMS has observed the decay of the Higgs boson into a pair of τ leptons with a significance of 5.9 s.d. [12], the $t\bar{t}H$ production mode at 5.2 s.d. [13]. The Higgs boson has also been seen in its decays into muons with a significance of 3 s.d. [14]. In the analyses of the decay channels $H \rightarrow \gamma\gamma$ and $H \rightarrow ZZ \rightarrow 4l$ the Higgs boson mass had been

measured to be 125.38 ± 0.14 GeV [15]. The width of the Higgs boson had been extracted and was found to be $\Gamma_H = 3.2^{+2.4}_{-1.7}$ MeV by using off-mass-shell and on-mass-shell Higgs boson production [16]. On-mass-shell refers to a particle with its physical mass, and off-mass-shell refers to a virtual particle.

Studies of on-shell and off-shell Higgs boson production in the four-lepton final state are presented in Ref. [17], using data from the CMS experiment (LHC) with the integrated luminosity of 80.2 fb^{-1} and center-of-mass energy of 13 TeV. Kinematic information from the decay particles and the associated jets are combined using matrix element techniques to identify the production mechanism and increase sensitivity to the H boson couplings in both production and decay. The constraints on anomalous HVV couplings were found to be consistent with the Standard Model expectation in both on-shell and off-shell regions. Under the assumption of a coupling structure similar to that in the Standard Model, the H boson width is constrained to be $3.2^{+2.8}_{-2.2}$ MeV while the expected constraint based on simulation is $4.1^{+5.0}_{-4.0}$ MeV.

Measurements of the Higgs boson mass in the diphoton decay mode at the LHC, with the 35.9 fb^{-1} and $\sqrt{s} = 13$ TeV data collected in 2016 was described in Ref. [18]. Although the branching fraction of the diphoton $H \rightarrow \gamma\gamma$ decay channel is significantly small ($\approx 0.23\%$), it provides a clean final state topology in which the diphoton invariant mass can be reconstructed with high precision.

A new method for estimating the systematic uncertainty associated with changes in the transparency of crystals in an electromagnetic calorimeter during radiation damage was also used. The value of the mass of the Higgs boson in the diphoton decay mode was found to be $m_H = 125.78 \pm 0.26$ GeV. This measurement had been combined with a recent result of the CMS of the same magnitude in $H \rightarrow ZZ \rightarrow 4l$ decay channel [19] to yield $m_H = 125.46 \pm 0.16$ GeV. Currently this is the most precise measurement of the mass of the Higgs boson.

The conclusions from the work done are the following: during Run 2 of the LHC the experimental collaborations started to employ the combined data for precision measurements of Higgs properties, such as mass, width, couplings, CP. All main production mechanisms are observed, including $H \rightarrow b\bar{b}H$, $t\bar{t}H$, VH and etc. Higgs boson mass m_h was measured with an accuracy of 0.1%. Precisions of cross section and branching ratio measurements in combined channel

were down to 8.5% level and had $\sim 6\text{-}30\%$ accuracy for measurements of couplings. The absolute value of a width was getting closer to the SM expectations (4.1 MeV). Scientists still need to improve the measurement accuracy. It should be noted that spin, parity, differential distributions were not contradict the SM.

There are strong theoretical motivations to search for CP-violating effects in couplings of the Higgs boson to fermions rather than V bosons. As is well known, the amount of CP violation in the SM encoded in the CP violating phase in the CKM matrix is not enough by about an order of magnitude to explain the matter-antimatter asymmetry in the Universe. In couplings to V bosons, CP-odd contributions may enter via nonrenormalisable higher-order operators that are suppressed by powers of $1/\Lambda^2$ [20-22]. Here Λ is considered as a physics scale beyond the SM. Therefore, we can assume that they will only make a small contribution to the coupling. A renormalisable Higgs-to-fermion coupling that violates the CP-symmetry may occur at tree level. The Yukawa couplings for τ lepton $H\tau\tau$ and t quark Htt are therefore the optimal couplings to study the CP in pp collisions [23], and measurements of these two interactions complement each other. Recently, both ATLAS [25] and the CMS [24] collaborations presented first measurements of the CP structure of the Higgs-to-top quarks coupling. The CMS results reject the purely CP-odd hypothesis with a significance of 3.2σ standard deviations, while the ATLAS results reject it with a significance of 3.9σ standard deviations.

The CP-properties of the $H \rightarrow \tau\tau$ decay is described in terms of an effective mixing angle $\alpha^{H\tau\tau}$, which is actually equal to 0° in the Standard Model [26]. It turns out that the study of the nonzero parameter $\alpha^{H\tau\tau}$ would directly contradict the SM predictions, and led to models beyond the SM: two-Higgs-doublet model [27], supersymmetry. For instance, according to the minimal supersymmetric model CP violation between the Higgs and fermion is expected to be small. Therefore the measurement of a sizeable mixing angle would be unfavorable for such scenarios. In contrast, the parameter $\alpha^{H\tau\tau}$ can be as large as 27° in the next-to-minimal supersymmetric model [28].

Parameterisation of the Lagrangian for the Yukawa t coupling in terms of the coupling strength modifiers k_τ and \tilde{k}_τ which parameterise the CP-even and CP-odd contributions, respectively [23]:

$$L_Y = \frac{-m_\tau}{v} H (k_\tau \bar{\tau}\tau + \tilde{k}_\tau \bar{\tau}i\gamma_5\tau).$$

In this equation, m_τ is the τ lepton mass, τ is the Dirac spinor for the τ lepton fields. The vacuum expectation value of the Higgs field v is taken as 246 GeV. The effective mixing angle $\alpha^{H\tau\tau}$ for the $H\tau\tau$ coupling is defined by the coupling strengths as

$$\tan(\alpha^{H\tau\tau}) = \frac{\widetilde{k}_\tau}{k_\tau}$$

while the fractional contribution of the CP-odd coupling $f_{H\tau\tau}^{CP}$ could be obtained from the mixing angle as $f_{H\tau\tau}^{CP} = \sin^2(\alpha^{H\tau\tau})$. A mixing angle of $\alpha^{H\tau\tau} = 0$ (90°) corresponds to a pure scalar (pseudoscalar) coupling. For any other value of the parameter $\alpha^{H\tau\tau}$, the Higgs boson has a mixed coupling with CP-even and CP-odd components, where the maximum mixing occurs at $\pm 45^\circ$.

ϕ_{CP} denotes the angle between two planes, where occur two τ -lepton decays in the rest frame of the Higgs boson. An illustration of the decay planes is shown in Figure 8. The relation between $\alpha^{H\tau\tau}$ and ϕ_{CP} can be derived as follows

$$\frac{d\Gamma}{d\phi_{CP}}(H \rightarrow \tau^+\tau^-) \sim 1 - b(E^+)b(E^-) \frac{\pi^2}{16} \cos(\phi_{CP} + 2\alpha^{H\tau\tau}).$$

In this equation, the energies of outgoing charged particles denoted by E^- and E^+ in their respective rest frames. The b functions are spectral functions that account the correlation between the τ -lepton spin and the momentum of the emitted charged particle.

In Figure 9 it is illustrated the normalized distribution of ϕ_{CP} for the pseudoscalar, scalar and maximally mixed values of $\alpha^{H\tau\tau}$, as well as the ϕ_{CP} distribution from Drell–Yan processes. Calculations was done in the rest frame of the Higgs boson.

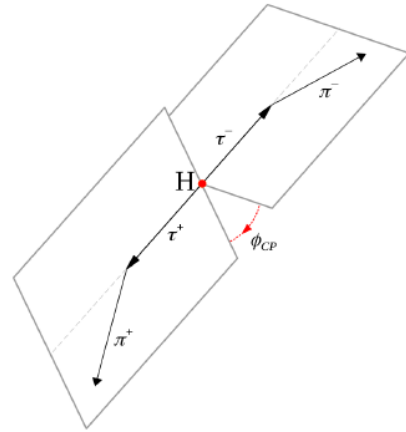


Figure 8 – The illustration of two τ lepton decay planes in the H rest frame. ϕ_{CP} denote the angle between this decay planes

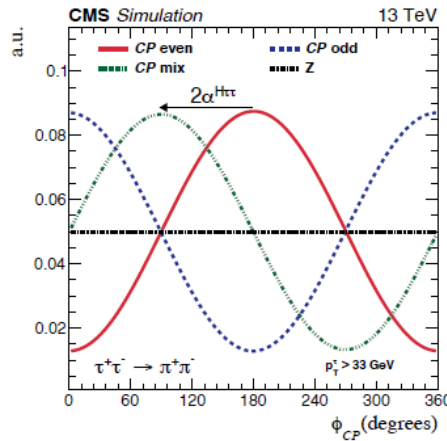


Figure 9 – The normalised distribution of ϕ_{CP} between the τ lepton planes in the rest frame of the Higgs boson at the generator level, for both τ leptons decaying to a charged pion and a neutrino. The distributions are for a decaying scalar (CP-even, solid red), pseudoscalar (CP-odd, dash blue), a maximal mixing angle of 45° (CP-mix, dash-dot-dot green), and a Z vector boson (black dashdot). The transverse momentum of the visible t decay products p_T was required to be larger than 33 GeV during the event generation

Results for the $\alpha^{H\tau\tau}$ mixing angle: the observed and expected negative log-likelihood scan for the combination of channels $\tau_e\tau_h$, $\tau_\mu\tau_h$, and $\tau_h\tau_h$ was

presented in Figure 10. The results disfavoured the pure CP-odd scenario at 3.0σ and expected exception assuming the SM Higgs boson is 2.6σ .

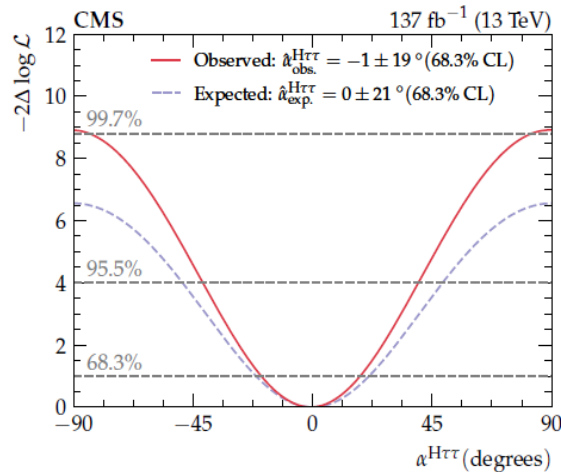


Figure 10 – Negative log-likelihood scan for the combination of the $\tau_e\tau_h$, $\tau_\mu\tau_h$, and $\tau_h\tau_h$ channels. The observed (expected) sensitivity to distinguish between the scalar and pseudoscalar hypotheses, determined at $\alpha^{H\tau\tau} = 0$ and $\pm 90^\circ$, respectively is $3.0s$ ($2.6s$). The observed (expected) value for $\alpha^{H\tau\tau}$ is $-1\pm 19^\circ$ ($0\pm 21^\circ$) at the 68.3% CL. The observed range is $\pm 41^\circ$ ($\pm 49^\circ$) at 95.5% CL and $\pm 84^\circ$ at the 99.7% CL

The first measurement of the CP structure of the Yukawa coupling between the Higgs boson and τ lepton is presented in Ref. [29]. The measurement was based on data collected in $p\bar{p}$ collisions at $\sqrt{s} = 13$ TeV by the CMS experiment at the LHC, with the integrated luminosity of 137 fb^{-1} . The analysis uses the angular correlation between the decay planes of τ leptons produced in Higgs boson decay. It is found that the observed effective mixing angle is $-1\pm 19^\circ$, while the expected value is $0\pm 21^\circ$ at a confidence level of 68.3%. The observed uncertainty found to be $\pm 41^\circ$ and expected uncertainty is $\pm 49^\circ$ at the 95.5% CL, and the observed sensitivity at the 99.7% confidence level is $\pm 84^\circ$. The main uncertainty in the measurement is statistical, which means that the accuracy of the measurement will increase as more collision data is accumulated. The result is consistent

with the predictions of the SM and reduces the allowed parameter space for its extensions.

In Ref. [30] it was studied the decay of the Higgs boson into four leptons via a virtual W-boson or Z-boson pair. This decay channel is one of the most important decay modes in the search

for the Higgs boson at the LHC. Partial decay widths of the $H \rightarrow 4 \text{ leptons}$ decays are given in Table 1 for various Higgs masses and various decay channels. The data includes the $\mathcal{O}(\alpha)$ and $\mathcal{O}(G_\mu^2 M_H^4)$ corrections and the corresponding relative corrections. The channel $\nu_e e^+ \mu^- \bar{\nu}_\mu$ results from the decay $H \rightarrow WW \rightarrow 4 \text{ leptons}$, while the last channel $\nu_l l^+ l \bar{\nu}_l$ receives contributions from both the decay into W and into Z bosons. The larger the Higgs mass, the larger is the decay width.

Table 1 – Descriptions of the distribution of the Higgs boson [30]

	$M_H(\text{GeV})$	140		170		200	
	$\Gamma_W(\text{GeV})$	2.09052...		2.09054...		2.09055...	
	$\Gamma_Z(\text{GeV})$	2.50278...		2.50287...		2.50292...	
H \rightarrow		$\Gamma[\text{MeV}]$	$\delta[\%]$	$\Gamma[\text{MeV}]$	$\delta[\%]$	$\Gamma[\text{MeV}]$	$\delta[\%]$
$e^+e^-\mu^+\mu^-$	corrected	0.0012628(5)	2.1	0.020162(7)	2.7	0.8202(2)	4.4
	lowest order	0.0012349(4)		0.019624(5)		0.78547(8)	
$l^+l^-l^+l^-$	corrected	0.0006692(2)	2.3	0.010346(3)	2.7	0.41019(8)	4.4
$l=e,\mu$	lowest order	0.0006555(2)		0.010074(2)		0.39286(4)	
$\nu_e e^+ \mu^- \bar{\nu}_\mu$	corrected	0.04807(2)	3.7	4.3109(9)	6.2	12.499(3)	5.0
	lowest order	0.04638(1)		4.0610(7)		11.907(2)	
$\nu_l l^+ l \bar{\nu}_l$	corrected	0.04914(2)	3.7	4.344(1)	6.1	14.133(3)	5.0
$l=e,\mu$	lowest order	0.04738(2)		4.0926(8)		13.458(2)	

because the available phase space grows. The corrections to partial decay widths typically amount to some per cent and increase with growing Higgs mass M_H , reaching about 8% at $M_H \sim 500\text{GeV}$. For not too large Higgs masses ($M_H \lesssim 400\text{GeV}$) the corrections to the partial decay widths can be reproduced within $\lesssim 2\%$ by simple approximations. The decays of the Higgs boson into 4 leptons via a W-boson or Z-boson pair lead to experimental signatures at the LHC that are important for studying the Higgs boson and its properties.

6 Conclusion

The SM predictions for different observables, such as widths and partial decay probabilities, various asymmetries, kinematic distributions, cross-sections of processes are in excellent agreement with experimental data. There are 18 free parameters in the SM:

- four parameters in the electroweak and Higgs sector: g_1, g_2, μ^2, λ . These parameters are expressed in terms of parameters $\alpha_{em} = \frac{e^2}{4\pi}, \sin\theta_W, M_Z, M_h$, which are measured in high accuracy;

- six quark masses and three charged lepton masses;

- in the Cabibbo-Kobayashi-Maskawa mixing matrix there are three angles and one phase;

- $\alpha_{QCD} = \frac{g_s^2}{4\pi}$ is the coupling constant of strong interactions.

The Higgs boson has been discovered and significant differences from the predictions of the Standard Model are not yet visible, but the physical characteristics of the Higgs boson is still to be studied in great detail. This is what physicists will continue to do at the LHC and at future high-energy colliders. In the new Run at LHC, a beam of protons appeared with a collision energy of up to 13.6 TeV, in addition, the luminosity, i.e. number of collisions have been increased. As expected, detectors will be able to see significantly more Higgs boson production events,

which means physicists will be able to study its properties in more detail and subject the Standard Model to even more rigorous tests. Run 3 will last approximately 3 years, after which the Large Hadron Collider will be radically upgraded to become the HL-LHC (High Luminosity Large Hadron Collider).

Also in the coming years, physicists must decide in which direction to develop particle physics in the near future. The LHC will be replaced by a new generation collider, and most likely it will be one of the variants of the Higgs factory – the electron-positron collider (FCC), optimized for the study of the Higgs boson. As a result of the work of the LHC and the Higgs factory, the measurement accuracy of many decays of the Higgs boson will improve to a few percent. If the properties of the Higgs boson differ from the Standard Model by even a few percent, this will be noticeable, and thus a new chapter in the study of the micro-world will open.

Let us note one more new generation project for further study of the Higgs sector and search for new physics beyond the SM. Such a project is the International Linear Collider (ILC), which proposes to collide e^+e^- at energies of 250 GeV and higher. The fundamental difference from the LHC is that leptons collide in the ILC, which makes the events cleaner to interpret. It is planned to install two conceptually different detectors at the ILC collider: ILD (International Large Detector) and SiD (Silicon Detector). Both detectors will have record-breaking accuracy in measuring the jet momentum and energies.

Acknowledgements

This work was supported by the Ministry of Education and Science of the Republic of Kazakhstan, Grant IRN AP09058640 “Physics beyond the Standard Model in processes involving the top quark and Higgs boson”.

The article is dedicated to the tenth anniversary of the discovery of the Higgs boson.

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Funded by Al-Farabi KazNU