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Revising the cosmological constant problem through a fluid different from the quintessence

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The role of a scalar field is reexamined through the introduction of a quasi-quintessence model. Unlike quintessence, in the quasi-quintessence model, the pressure of the scalar field does not depend on its kinetic energy. This research note provides an overview of the main characteristics of this scenario and outlines how cosmic fluids can be constructed based on quasi-quintessence. A significant application of quasi-quintessence is presented, starting with a reference to Weinberg's no-go theorem, related to the cosmological constant problem. By assuming the occurrence of a phase transition induced by a fourth-order quasi-quintessence potential and suggesting that the violation of the no-go theorem happens exclusively during this phase transition, it is possible to argue a mechanism for addressing the cosmological constant problem. This mechanism involves a form of vacuum energy cancellation, offering a potential solution to the long-standing cosmological constant problem. The discussion also delves into the avoidance of fine-tuning adjustments and explores the implications of this approach in the realms of dark energy and inflation. Physical consequences of the quasi-quintessence scenario are presented, shedding light on its potential benefits and drawbacks.

Key words: cosmology, early universe, scalar fields, dark matter, dark energy.
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1 Introduction

In the standard cosmological puzzle, the universe's evolution is described by a combination of various components, including pressureless baryonic matter, radiation, neutrinos, dark matter, and dark energy. Each of these components is represented as a cosmic fluid with well-defined equations of state.

Among these components, dark energy is particularly intriguing as it possesses a negative pressure that is believed to be responsible for the current acceleration of the universe. However, the true nature of dark energy remains a mystery, and it is one of the outstanding challenges in modern cosmology. Similarly, dark matter, another enigmatic component, also defies direct detection and identification within the context of general relativity.

In recent years, alternative cosmological models and theories have emerged as potential solutions to the puzzles posed by dark energy and dark matter [1]. These alternatives range from modifications of Einstein's theory of gravity to more complex equations of state or unconventional models that challenge the standard cosmological paradigm. These diverse approaches reflect the ongoing quest to understand the fundamental properties of the universe and the nature of its mysterious constituents.

Interestingly, *unified dark energy* models represent intriguing approaches to address both the dark energy and dark matter by introducing a *single cosmic fluid* that exhibits different behaviors on various cosmological scales. Among the proposed unified dark energy models, the concept of a "dark fluid" stands out as one of the simplest ones [2]. It is entirely consistent with the standard ΛCDM paradigm and can be conceptually visualized as a fluid with constant pressure but varying density and equation of state.

To theoretically realize the dark fluid scenario, different strategies have been explored. For example, one approach involves setting the adiabatic index to zero for a specific barotropic fluid, while another approach utilizes a purely adiabatic fluid where the sound speed naturally becomes zero [3].

While the concept of a dark fluid with constant pressure aligns with the standard cosmological model, it still leaves questions about its fundamental nature and microphysics.

A promising scenario that makes use of scalar fields and heals the cosmological constant problem, is offered by quasi-quintessence.

Quasi-quintessence has emerged as scalar field exhibiting pressure solely dependent on its potential, i.e., decoupled from kinetic energy. In this way, the quasi-quintessence approach differs from quintessence because the pressure does not depend on kinetic degrees of freedom and so the corresponding sound speed identically vanishes [4].

In this research note, I provide an overview of quasi-quintessence, outlining its fundamental characteristics and how it can be derived from a Lagrangian formulation. I also discuss and compare previous studies related to quasi-quintessence, summarizing their key findings. Further, I delve into specific cases and scenarios associated with quasi-quintessence, shedding light on its role in modeling matter-like fluids with non-zero pressure. Additionally, I highlight some practical applications of quasi-quintessence within the realms of dark energy and inflation, showcasing its versatility in addressing significant cosmological questions. Finally, I touch upon the connections between quasi-quintessence and the cosmological constant problem, underlining its potential to offer novel insights into this long-standing challenge in cosmology.

The paper is structured as follows. In Sect. II, I discuss how to obtain quasi-quintessence and how it can be obtained from different procedures. In Sect. III, peculiar cases of interest are reported, showing how quasi-quintessence mimes different fluids and comparing this fact with quintessence. The role of matter-like fluid is discussed and then in Sect. IV, I develop dark energy and inflation in terms of quasi-quintessence. The connections with the cosmological constant problem have been summarized in Sect. V.

2 Quasi-quintenssece

A possible fundamental representation of quasi-quintessence involves the Lagrangian [4]

$$\mathcal{L}^{QQ} = K(X,\phi) + \lambda Y[X,\nu(\phi)] - V(\phi), \quad (1)$$

where

– $K(X,\phi)$ and $V(\phi)$ are the usual kinetic part and potential respectively, with the remark that K depends on $X \equiv \frac{1}{2} g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$ and so can be seen as a generalized kinetic contribution;

– λ represents a Lagrange multiplier which ensures to constrain the overall universe energy and is associated with the non-dynamical function Y. Manifestly, the function $\nu(\phi)$ dictates the scalar field's inertial mass.

Clearly, quintessence can be fully-described by setting the Lagrange multiplier to zero and allowing $K \to X$.

The energy-momentum tensor reads

$$T_{\alpha\beta} = 2X\mathcal{L}_{,X}v_\alpha v_\beta - (K - V)g_{\alpha\beta}, \quad (2)$$

where we identified the thermodynamic quantities:

$$\rho = 2X\mathcal{L}_{,X} - (K - V), \quad (3a)$$

$$P = K - V, \quad (3b)$$

$$v_\alpha \equiv \partial_\alpha\phi/\sqrt{2X}, \quad (3c)$$

being the density, the pressure and the effective 4-velocity, respectively.

At this point, it seems no differences occur between Eqs. (3) and the quintessence case, but rather a simple generalization of it.

Therefore, the Lagrange multiplier and the generalized kinetic term are meant to play a crucial role in distinguishing quasi-quintessence from quintessence.

For the sake of completeness, without its inclusion, there is no possibility of obtaining a quasi-quintessence field from a fundamental Lagrangian, as the pressure would feature a kinetic contribution.

The main distinctions between quintessence and quasi-quintessence fields lie in the form of pressure and consequently on the sound speed, $c_s \equiv \sqrt{\frac{\partial P}{\partial \rho}}$.

Specifically, the sound speed for quintessence, c_s^Q, and quasi-quintessence, c_s^{QQ}, are

$$c_s^Q = 0, \quad (4a)$$

$$c_s^{QQ} = 1, \quad (4b)$$

showing that quasi-quintessence does not provide perturbation speed, resembling dust. This property will be referred to as a matter-like fluid later in the text.

A. Alternative derivations

We here focus on alternative strategies to reproduce quasi-quintessence. While we report such results, we emphasize the main drawbacks in determining a quasi-quintessence fluid from alternative derivations different from Eq. (1).

1. The first derivation consists in directly modifying the energy-momentum tensor. In this approach, there is no need for a Lagrange multiplier, and the modification is described by the equation [5]:

$$T_{\mu\nu} \to T_{\mu\nu} + V(\phi) g_{\mu\nu} - X g_{\mu\nu}. \qquad (5)$$

Changing the energy-momentum tensor fine tunes the structure of density and pressure and so this approach turns out to be appealing for its simplicity but clearly suffers from the lacking of a Lagrangian description. Hence, it may somehow appear unphysical from a fundamental viewpoint.

2. The second approach consists in exotic quintessence. Here, assuming that two fluids are interacting, with two different equations of state, w_1 and w_2, we get $3H^2 = \rho_1 + \rho_2$, $\dot{\rho_1} + \dot{\rho_2} + 3H((1 + w_1)\rho_1 + (1 + w_2)\rho_2 = 0$. If the interaction is modeled by a scalar field, ϕ, we have [6]

$$\dot{\phi}^2 = (1 + w_1)\rho_1 + (1 + w_2)\rho_2, \qquad (6)$$

with $\dot{\phi}^2 = -2\dot{H}$, whose dynamics is

$$\ddot{\phi} + \frac{3}{2}(1 + w_1)H\dot{\phi} + \frac{w_1 - w_2}{2}\frac{\dot{\rho_2}}{\dot{\phi}} = 0. \qquad (7)$$

Direct integration leads to $\dot{\rho_2} + A\dot{\phi}\rho_2 = 0$, where A is a constant. After cumbersome algebra, we find that the energy density of the second fluid can be associated with an exponential potential $\rho_2 = \rho_{20} H_0^2 e^{-A(\phi-\phi_0)} = V(\phi)$,where ρ_{20} is a positive integration constant and H_0, ϕ_0 are the present values of Hubble constant and scalar field, respectively.

Hence, the density and pressure become

$$\rho = \frac{\dot{\phi}^2}{1 + w_1} + \frac{w_1 - w_2}{1 + w_1} V, \qquad (8)$$

$$p = w_1 \frac{\dot{\phi}^2}{1 + w_1} - \frac{w_1 - w_2}{1 + w_1} V, \qquad (9)$$

$$\ddot{\phi} + \frac{3}{2}(1 + w_1)H\dot{\phi} + \frac{w_1 - w_2}{2}\frac{dV}{d\phi} = 0, \qquad (10)$$

Now, **including two extra fluid components**, with energy densities ρ_3 and ρ_4, **all the components of Einstein equations yield**

$$3H^2 = \rho_1 + \rho_2 + \rho_3 + \rho_4, \qquad (11)$$

$$\dot{\rho_1} + \dot{\rho_2} + 3H[(1 + w_1)\rho_1 + (1 + w_2)\rho_2] = 0, (12)$$

$$\dot{\rho_3} + 3H(1 + w_3)\rho_3 = 0, \qquad (13)$$

$$\dot{\rho_4} + 3H(1 + w_4)\rho_4 = 0, \qquad (14)$$

Analogously to the case of two fluids, the scalar field here depicts the interacting term between ρ_1 and ρ_2.

The total energy density and pressure of the four-fluid mixture, and the dynamical equation for the scalar field, with the **arbitrary ansatz** $w_1 = w_3 = 0$ and $w_4 = 1/3$, are

$$\rho = \dot{\phi}^2 - w_2\rho_2 + \rho_3 + \rho_4, \qquad (15a)$$

$$p = w_2\rho_2 + w_3\rho_3 + w_4\rho_4, \qquad (15b)$$

$$\ddot{\phi} + \frac{3}{2}H\dot{\phi} - \frac{w_2}{2}\frac{\dot{\rho_2}}{\dot{\phi}} = 0, \qquad (15c)$$

that appear very close to the quasi-quintessence scenario since ρ_2 is integrated out providing the dependence on ϕ, provided that $\phi \to \phi/\sqrt{2}$ and $-w_2\rho_2 \to V(\phi)$.

Even though this approach is formally equivalent to quasi-quintessence, it fails to involve one fluid representation as in the case of the fundamental Lagrangian presented in Eq. (1). Hence, the exotic quintessence is not physically equivalent to quasi-quintessence and, again, the present derivation fails to be predictive.

3 The role of Lagrange multiplier

Lagrange multipliers are a useful tool for incorporating constraints into complicated problem. Although their name may be misleading, these multipliers indeed carry a physical interpretation as well. It is essential to be cautious when adding constraints without considering their physical implications, as doing so can result in an incorrect solution.

This model exhibits similarities to the concept of a dark fluid, which is characterized by a constant pressure, varying density, and equation of state.

So, we can summarize the following results

$$\begin{aligned} w_Q \neq 0 \quad c_{s,Q} = 1, \\ w_{QQ} \neq 0 \quad c_{s,QQ} = 0, \\ w_m = 0 \quad c_{s,m} = 0, \end{aligned} \tag{16}$$

where pure matter, with subscript m implies zero equation of state and sound speed, while the quasi-quintessence fluid appears closer to the matter definition, because it does not show a stiff matter behavior in c_s.

Thus, it appears evident that,

– if $K = 0$, $\lambda \neq 0$, $X \neq 0$ and $V = 0$, we reproduce baryons,

– if $K = const$, $\lambda \neq 0$, $X \neq 0$ and $V \neq 0$, we obtain matter with pressure, or *quasi-dust*,

– if $K \equiv X$, $\lambda = 0$, and $V \neq 0$, we have stiff matter, equivalent to quintessence.

So, the trivial solution corresponds exclusively to baryons under the form of dust.

On the other hand, if $K = K_0 = const$ and $\lambda = 0$, then $\mathcal{L}_X = 0$, and $P/\rho = -1$ always, meaning that the quasi-quintessence formulation cannot be applied to a Lagrangian comprising only kinetic and potential terms. When I will discuss on how to alternatively derive quasi- quintessence, this argument will become central.

However, if $K = const$ and $\lambda \neq 0$, then $\mathcal{L}_X = \lambda Y_X$, and in fact Eqs. (3) become

$$\rho = 2X\mathcal{L}_{,X} + \mathcal{V}(\phi), \tag{17}$$

$$P = -\mathcal{V}(\phi), \tag{18}$$

where $\mathcal{V}(\phi) \equiv V - K_0$, with $\mathcal{L}_X = \lambda Y_X$.

Now let us consider quintessence, where by definition has $\lambda = 0$ and $K_X = 1$, we would obtain

$$w = -\frac{1}{1 - \frac{2X}{X - V}}. \tag{19}$$

So, if $\phi = \phi_0$, then $X = 0$, implying

$$w = -1 \quad c_s = 1 \tag{20}$$

thus, quintessence in a constant value of the potential implies always stiff matter perturbations with cosmological constant equation of state.

Conversely, a dust-like fluid by definition has $\lambda \neq 0$ and $K \neq X$. We would obtain

$$w = -\frac{1}{1 - \frac{2X(K_X + \lambda Y_X)}{K - V}}, \tag{21}$$

where in the simplest case, $K = X$, it becomes $w = -1$ at $\phi = \phi_0$, as evident from

$$w = -\frac{1}{1 - \frac{2X(K_X + \lambda Y_X)}{X - V}}. \tag{22}$$

An interesting case occurs as$V = 0$, having that different fluids can be obtained by virtue of fixing λ and Y_X appropriately. Below, we can focus on some relevant cases.

– **Stiff matter-like.** In the quasi-quintessence picture, it is possible to obtain a generic fluid when $V = 0$, having

$$w^{QQ} = -\frac{1}{1 - \frac{2X(K_X + \lambda Y_X)}{K}}, \tag{23}$$

that is not stiff matter because it does not happen simultaneously that $c_s = 0$ an $w = 1$ with arbitrary choices of X, K and Y. In the quintessence scenario, on the other hand, if $V = 0$, the fluid is always stiff matter, since $c_s = 1$ and $w^Q = 1$ are easily fulfilled at the same time. For quasi-quintessence, we obtain stiff matter when

$$X(K_X + \lambda Y_X) = K. \tag{24}$$

If $Y_X = 0$, then $XK_X = K$, yielding

$$K = \beta X, \tag{25}$$

with β unspecified constant. If $Y_X \neq 0$, the solution, for constant Y_X, becomes

$$K = \beta X + \lambda Y_X X. \tag{26}$$

– **Generic fluids.** Motivated by Eq. (25) and (26), we can select the peculiar case $K = X$, leading to

$$w = \frac{1}{1 + 2\lambda Y_X}. \tag{27}$$

Hence, $\lambda Y_X \in R^+$ and $\lambda Y_X \in R^-$ give rise to standard and exotic fluids respectively, for some values of Y_X. Nevertheless, assuming Y_X fixed, $w = w(\lambda)$ i.e., implying that the equation of state depends on the peculiar choice of λ and can have different alternative values.

– **Matter-like fluids.** In quasi-quintessence, it is not possible for w to be zero unless trivial cases are considered. Specifically, unphysical cases with divergent values for X, K, and K_X would imply $w \to 0$. However, if $K = \mathcal{V} = 0$ with $X \neq 0$ and $\lambda Y_X \neq 0$, it can result in $w = 0$. More generally, if $X = X_0$ and $V = V_0$ and $K_0 = V_0$, we obtain the same result of $w = 0$. In this case, K_0 and V_0 can be constants since K and V are functions of independent variables, ϕ and $\partial\phi$, so they cannot be equal for any value of ϕ, i.e., $K = V$ only if $K = V = 0$.

In this scenario, both $w = 0$ and $c_s = 0$ simultaneously, corresponding to baryons. Interestingly, this holds true for both cases where $K = X$ and $K \neq X$. Remarkably, the same outcome occurs in quintessence.

In quasi-quintessence, it is possible for $X = X_0$ and $V = V_0$ to coexist without resulting in dust. Assuming that $K = K_0$ due to $X = X_0$, it is evident that $K_0 \neq V_0$ can prevent w from becoming zero. In this regard, quasi-quintessence exhibits a broader range of possibilities compared to quintessence.

– **The cosmological constant.** A cosmological constant has $w = -1$ and $c_s = 0$. So, arguing a cosmological constant in quasi-quintessence implies to find out a mathematical way to construct the cosmological constant without plugging it by hands into the energy-momentum tensor.

An immediate focus on quintessence could be instructive before focusing on quasi-quintessence. There, $V \gg X$ yields $w \simeq -1$. Even though appealing, this mimics the cosmological constant regime, but still preserves $c_s = 1$. So, in quintessence the cosmological constant cannot be obtained with arbitrary X unless one either approximates it as above or sets $X = 0$.

In quasi-quintessence, instead, we have cosmological constant if

$$XK_X + \lambda XY_X = 0, \tag{28}$$

so, for constant Y_X

$$K = -\lambda Y_X X + K_0, \tag{29}$$

whose limit, $\lim_{X\to 0} K = K_0$, is not necessarily zero, i.e., K does not forcedly vanish as $X \to 0$.

Conversely, $X = 0$ implies $K = V$, in analogy to quintessence.

So, besides the fact that the cosmological constant is reachable in quasi-quintessence, the most relevant fact consists in reproducing the very well-known slow-roll phase, consisting here in $V \ll K$ that does not reproduce $w \simeq -1$ since

$$w = -\frac{V}{V + 2X(K_X + \lambda Y_X)}, \tag{30}$$

that interestingly is not phantom, i.e., $w > -1$, as

$$\frac{X(K_X + \lambda Y_X)}{V + 2X(K_X + \lambda Y_X)} > 0. \tag{31}$$

While quasi-quintessence is a dust-like fluid with pressure, the traditional dust remains pressureless. Since the fluid behaves like matter, exhibiting a pressure, but it is not dust, we can assume that its nature is associated with dark matter. For examples of dark matter with pressure, see e.g., Ref. [4] and references therein.

This assumption has significant implications. Firstly, the equation of state for dark matter cannot be zero; it must have a non-zero value. Secondly, since this dark matter component is believed to contribute to the universe's current acceleration, its equation of state should be negative.

As a result, the equation of state for dark matter becomes exotic, consisting of a quasi-quintessence fluid that plays a role in accelerating the universe today. This implies a unified dark energy model where the same fluid acts as both dark matter and dark energy. Consequently, the cosmological model based on quasi-quintessence can be viewed as a unification model. Although it appears similar in form to the ΛCDM model, it is reinterpreted as a model featuring dark fluid.

4 The cosmological constant problem

In this subsection, we explore the concept of a "self-adjustment" Lagrangian, which could potentially offer a solution to the cosmological constant problem. The underlying idea involves the incorporation of a matter-like sector, similar to what quasi-quintessence achieves, with the specific role of mitigating the influence of the large vacuum energy.

A. Limitations from the Weinberg's no go theorem

To do so, it is essential to recall the Weinberg's no-go theorem, according to which any self-adjustment mechanism is jeopardized by fine-tuning. To show that, given the Lagrangian, $\mathcal{L}[g,\varphi_i]$ we require that [7]

– the vacuum is "translationally invariant", implying that on-shell, we have $g_{\mu\nu},\varphi_i = const$;

– the corresponding symmetry is GL(4);

– the vacuum expectation value of the overall Lagrangian might be constant before and zero after the adjustment. This is necessary to ensure the cancellation of the cosmological constant.

Assuming $M^{\mu}{}_{\nu}$ is a constant 4 × 4 matrix, before the adjustment, a constant vacuum solution occurs, say

$$\delta_{\delta M}\,\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\varphi_i}\delta_{\delta M}\varphi_i + \frac{\partial\mathcal{L}}{\partial g_{\mu\nu}}\delta_{\delta M}g_{\mu\nu}. \quad (32)$$

On the other hand, the vacuum field equations, as expected in the end of the adjustment,

$$\frac{\partial\mathcal{L}}{\partial\varphi_i} = 0, \quad \frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = 0. \quad (33)$$

So, to guarantee the latter relations to occur from Eq. (32), either $\frac{\partial\mathcal{L}}{\partial\varphi_i} = 0$ alone regardless $\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}}$, or both are zero at the same time. For the first scenario, $\frac{\partial\mathcal{L}}{\partial\varphi_i} = 0$, $\delta_{\delta M}\,\mathcal{L}$ implies

$$\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}}\left(\delta M_{\mu\nu} + \delta M_{\nu\mu}\right) = Tr\delta M\mathcal{L} \Rightarrow \frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = \frac{1}{2}g^{\mu\nu}\mathcal{L} \quad (34)$$

that, once solved, provides

$$\mathcal{L} = \sqrt{-g}V(\varphi_i) \quad (35)$$

and so, to guarantee that $\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = 0$ holds, $V(\varphi_i) = 0$ is the unique solution. This clearly induces a severe fine-tuning.

On the other side, following the original formulation made by Weinberg, assuming that Eqs. (33) do not hold independently we write

$$2g_{\mu\nu}\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = \sum_i f_i(\varphi)\frac{\partial\mathcal{L}}{\partial\varphi_i}, \quad (36)$$

representing an appropriate formulation of the problem in which we introduce weights, f_i, depending on how the self-adjusting fields work.

The above relation suggests a new scaling symmetry, namely $\delta_\epsilon g_{\mu\nu} = 2\epsilon g_{\mu\nu}$, $\delta_\epsilon\varphi_i = -\epsilon f_i$, that under transformations, φ_i by $\varphi_i \to \tilde{\varphi}_i$, and $\delta_\epsilon g_{\mu\nu} = 2\epsilon g_{\mu\nu}, \delta\tilde{\varphi}_0 = -\epsilon, \delta\tilde{\varphi}_{i\neq 0} = 0$, and because $\delta_\epsilon\left(e^{2\tilde{\varphi}_0}g_{\mu\nu}\right) = 0$, end up with

$$\mathcal{L} = \mathcal{L}\left(e^{2\tilde{\varphi}_0}g_{\mu\nu},\tilde{\varphi}_{i\neq 0}\right). \quad (37)$$

The transformations turn into

$$\delta_{\delta M}\,\mathcal{L} = \frac{\partial\mathcal{L}}{\partial\tilde{\varphi}_0}\delta_{\delta M}\tilde{\varphi}_0 + \frac{\partial\mathcal{L}}{\partial\tilde{\varphi}_{i\neq 0}}\delta_{\delta M}\tilde{\varphi}_{i\neq 0} + \frac{\partial\mathcal{L}}{\partial g_{\mu\nu}}\delta_{\delta M}g_{\mu\nu}, \quad (38)$$

yielding

$$\frac{\partial\mathcal{L}}{\partial\tilde{\varphi}_{i\neq 0}} = 0, \quad \frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = 0, \quad (39)$$

where $\tilde{\varphi}_0$ since it is assumed to be the only scalar.

In a manner analogous to our previous approach, we assume that $\frac{\partial\mathcal{L}}{\partial\tilde{\varphi}_{i\neq 0}} = 0$ without employing $\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}}$. By using our expressions for $\delta_{\delta M}\,\mathcal{L}$, we derive the solution:

$$\mathcal{L} = \sqrt{-\det\left(e^{2\tilde{\varphi}_0}g_{\mu\nu}\right)}\,V(\tilde{\varphi}_{i\neq 0}) = \sqrt{-g}e^{4\tilde{\varphi}_0}V(\tilde{\varphi}_{i\neq 0}). \quad (40)$$

Consequently, from $\frac{\partial\mathcal{L}}{\partial g_{\mu\nu}} = 0$ we deduce $e^{4\tilde{\varphi}_0}V(\tilde{\varphi}_{i\neq 0}) = 0$, which leads to the only physical possibility, namely $V(\tilde{\varphi}_{i\neq 0}) = 0$. Again, this situation introduces a thorny fine-tuning.

Hence, any proposal aimed at resolving the cosmological constant problem must address how it circumvents the need for such fine-tuning.

B. Inflationary phase transition

There is no consensus in overcoming the Weinberg no go theorem. Here, however, we report one possible scenario that involves a constant vacuum solution comprising two components: one based on the potential contribution and the other on the kinetic part, which must be frozen and become constant. Immediately, assuming that the kinetic contribution is approximately constant one can recall quasi-quintessence.

In this context, if the potential is not canceled, say it is not adjusted, but is transformed into a chemical potential as due to particle production during a metastable phase, the fine-tuning issue can be mitigated and the no go theorem can be healed. To clarify it, we intend not to resolve the no go theorem, but rather, we explain why the potential is canceled and how the energy density is transformed, violating it.

Accordingly, the only viable approach to circumvent the no-go theorem is to introduce a metastable phase that violates the theorem as stated. During this phase, the corresponding GL(4) symmetry is broken, briefly disrupting the translational invariance of the vacuum.

During this short-lived phase, particles can be created from vacuum energy. The adjustment mechanism modifies the magnitude of the constant violating the no-go theorem only during a phase transition, which is not a stable state but a temporary deviation from the no-go theorem's constraints.

Hence, let us introduce the simplest Lagrangian for phase transition through the potential embedded in a thermal bath [8]

$$V(\phi) = V_0 + \frac{\chi}{4}\phi_0^4 + \frac{m_{eff}^2}{2}\phi_0^2\phi^2 + \frac{\chi}{4}\phi^4, \qquad (41)$$

where the symmetry breaking occurs as the effective mass changes sign. Here, χ is a dimensionless coupling constant and ϕ_0 represents the value of ϕ at the minimum.

As a result, the critical temperature induces a phase transition with significant consequences:

– Before the transition, $T > T_C$: The minimum associated with the potential lies at $\phi = 0$, and the potential reads $V_0 + \frac{\chi}{4}\phi_0^4$.

– During the transition: The universe experiences an inflationary phase with vacuum energy acting as a source.

– After the transition, $T < T_C$: When the temperature drops below T_C, the minimum of the potential shifts to $\phi = \phi_0$, and the potential is given by V_0 at $\phi = \phi_0$.

As a byproduct of the above the classical cosmological constant problem is revised if we set $V_0 = -\frac{\chi}{4}\phi_0^4$, then vacuum energy density, denoted as ρ_{vac}, becomes zero before the transition, implying that ρ_{vac} is non-zero after the transition. Conversely, if we choose $V_0 = 0$, vacuum energy is set to zero after the transition, but before the transition, ρ_{vac} is non-zero.

In both cases, it is crucial to note that vacuum energy cannot be zero simultaneously before and after the transition because the offset V_0 cannot vanish during those periods.

However, by revisiting the classical cosmological constant problem with quasi-quintessence, as demonstrated in [9], it is possible to employ a constant generalized kinetic term. This is achieved by:

1. Ensuring shift symmetry invariance, namely $\phi \to \phi + c_0$, with c_0 as a generic constant.
2. Fixing the conserved currents through the Noether theorem.
3. Requiring structures to form at all scales and invoking standard thermodynamics.

Meeting these requirements incorporates the sign of K_0 as the opposite of the potential at its minima. As the two signs are opposite, we can potentially provide a resolution to the cosmological constant problem.

In summary, quasi-quintessence offers a novel approach to addressing the cosmological constant problem by introducing a constant kinetic term and shift symmetry invariance, providing an alternative description of dark energy.

Bearing this in mind, we thus re-explore below the two possibilities to fix the offset and delete the cosmological constant.

1) If we select $V_0 = -\chi\varphi_0^4/4$, then before the transition we obtain $V = 0$ and therefore

2)

$$P_1 = \begin{cases} K_0, & (BT) \\ K_0 + \chi\varphi_0^4/4, & (AT) \end{cases}, \qquad (42)$$

$$\rho_1 = \begin{cases} 2X\lambda Y_X - K_0, & (BT) \\ 2X\lambda Y_X - K_0 - \chi\varphi_0^4/4, & (AT) \end{cases}, \qquad (43)$$

where we labeled the pressure and density with the subscript "1" to indicate that we are employing the first possible case. Here, we have in addition that K_0 may turn into $K_0 < -\chi\varphi_0^4/4$.

3) If we select $V_0 = 0$, after the transition we find $V = 0$, so that

$$P_2 = \begin{cases} K_0 - \chi\varphi_0^4/4, & (BT) \\ K_0, & (AT) \end{cases}, \quad (44)$$

$$\rho_2 = \begin{cases} 2X\lambda Y_X - K_0 + \chi\varphi_0^4/4, & (BT) \\ 2X\lambda Y_X - K_0, & (AT) \end{cases}, \quad 45)$$

where again we labeled the pressure and density with the subscript "2" to indicate the second occurrence, having this time $K_0 < 0$.

Consequently, in both cases $K_0 < 0$, albeit their magnitude upper values change accordingly. Even though the model offers a possibility in circumventing the no go theorem and the cosmological constant problem, the nature of the potential during the phase transition, its compatibility with Planck measurements on inflationary potential, and its behavior in small and

large field domains are however not well-established and require further investigation.

5 Late and early dynamics: dark energy and inflation

In view of the above results, let us focus on dark energy first.

The cosmological constant contribution is generally defined as [10]

$$\Lambda = \Lambda_B + \Lambda_{vac}, \quad (46)$$

where Λ_{vac} is the value of vacuum energy associated with quantum fluctuations, whereas Λ_B is the bare cosmological constant that enters the Einstein's field equations in fulfilment with the Bianchi identities. In our picture, above, $\Lambda_{vac} \simeq \frac{\chi}{4}\phi_0^4$.

Then, after the transition phase, Λ_{vac} is deleted as we showed in the two mentioned cases earlier. The only remaining contribution is the bare cosmological constant that, after the transition, induces the universe to accelerate when it comes dominant over matter and radiation.

The fine-tuning is avoided because the bare cosmological constant is the only quantity that is proportional to the trace of the Einstein equations and the field, φ, is no longer dynamical.

The bare cosmological constant plays the role of *emergent cosmological constant*, which is negligible with respect to the vacuum energy.

While the fine-tuning problem is clearly removed, because the high value of the predicted vacuum energy density is suppressed, the coincidence problem remains. To alleviate it, let us recall that the bare cosmological constant magnitude might be proportional to the trace of the Einstein tensor. Thus, it corresponds to the sum between $\rho + 3P$ of all the species filling the universe.

The coincidence problem is therefore fixed assuming that the constant remaining after the phase transition is a bare cosmological constant associated with the trace of the Einstein tensor.

Bearing this in mind, the Hubble parameter, $H(z)$, becomes

$$H(z) \equiv H_0 \sqrt{\frac{2X\lambda_0 Y_X}{\rho_{c,0}}(1+z)^3 + \frac{K_B}{\rho_{c,0}}}, \quad (47)$$

where we can identify the bare contribution induced into the definitions of mass and dark energy densities, $\Omega_m \equiv 2X\lambda_0 Y_X/\rho_{c,0}$, $\Omega_\Lambda \equiv -K_B/\rho_{c,0}$, whereas λ_0 is the initial value of the Lagrange multiplier.

About inflation, during the transition, where the no-go theorem is violated, inflation becomes a crucial aspect of the cosmological model. This inflationary phase is driven by the violation of the theorem and involves the contribution of vacuum energy, leading to a de Sitter-like accelerating expansion of the universe.

To describe this transition accurately, the potential must have a more intricate form than a simple fourth-order potential. This is because the fourth-order potential has been ruled out as a candidate for inflationary potentials based on observations, such as those from the Planck mission [11].

The key requirements for constructing a potential that characterizes the metastable phase during the transition are as follows:

– *Inducing phase transition*: The potential should induce a phase transition. In simpler terms, its behavior for small field values should resemble something proportional to φ^4.

– *Transition end*: The universe must exit the transition phase and settle into the symmetry minimum once the metastable phase ends.

To meet these requirements, several considerations come into play:

– *Role of the field*: The field itself induces the phase transition, and it acts as the inflaton during the metastable phase.

– *Vacuum energy suppression*: The cancellation mechanism via the quasi-quintessence field suppresses vacuum energy during the transition, which is essential for resolving the cosmological constant problem.

– *Similarities to old and new inflation*: The described mechanism bears similarities to both old and new inflation scenarios. It involves a phase transition like old inflation but has the potential to escape the metastable phase in a manner akin to new inflation.

The goal is to unify the key features of old and new inflation into a single framework, where new inflation is induced by the mechanisms that violate the no-go theorem during the transition.

As inflation progresses, the potential can reach a fixed point, and it is assumed that the phase transition occurs due to vacuum energy. This transition halts when an attractor state is reached, characterized by conditions where the field approaches its minimum value φ_0 and its time derivative $\dot{\varphi}$ approaches zero.

After the transition, the behavior of vacuum energy changes according to this scheme. It is likely transformed into particles created during the reheating phase or through particle production related to the evolving geometry of the universe.

Therefore, a suitable potential for this scenario should be constructed with the following considerations in mind:

– *Smooth transition:* The potential should smoothly connect the phases before and after the transition, ensuring continuity in the energy budget of the universe.

– *Compatibility with observations:* The potential should align with current observational constraints, such as those from the Planck mission.

– *First-order phase transition:* The thermodynamics of the potential should exhibit a first-order phase transition.

By addressing these aspects, we consider a positive-definite general double-exponential potential [4]

$$V(\Phi) = V_0 + \\ +\mathcal{A}[a\exp(\alpha\Phi^{c_1}) + +b\exp(\beta\Phi^{c_2})]^m, \qquad (48)$$

where $\Phi \equiv \varphi/\varphi_0$ is clearly the quasi-quintessence field normalized over its value φ_0 at the minimum of the potential and the arbitrary constants $\mathcal{A}, a, b, \alpha, \beta, c_1, c_2$ and m can be fixed in a general and unrestricted manner. For example, requiring $V(0) = V_0 + \chi\varphi_0^4/4$ at $\Phi = 0$ (or $\varphi = 0$), implies that $a = -b\exp(\beta - \alpha)$, and furthermore for $\Phi = 1$, or $\varphi = \varphi_0$, we must have $V(1) = V_0$, leading to $\mathcal{A} = \left(\frac{\chi\varphi_0^4}{4}\right)[b - b\exp(\beta - \alpha)]^{-m}$. Thus, for $\beta = 0$, $\alpha < 0$ and $c_1 = 1$, and $m = 2$, we obtain

$$V_1(\Phi) = V_0 + \frac{\chi\varphi_0^4}{4}\left[\frac{1 - e^{-|\alpha|(\Phi-1)}}{1 - e^{|\alpha|}}\right]^2, \qquad (49)$$

where m was fixed since, in the regime of small oscillations around $\Phi = 1$, the potential is expected to be quadratic [12].

The so-obtained potential mimes the Starobinsky potential $V(\Phi) = \Lambda^4\left[1 - e^{-\sqrt{\frac{2}{3}}\left(\frac{\varphi_0}{M_{Pl}}\right)\Phi}\right]$ for

$$\Lambda^4 \simeq \frac{\chi\varphi_0^4}{4}\left(1 - e^{|\alpha|}\right)^{-2}, \quad \alpha \simeq \sqrt{\frac{2}{3}\frac{\varphi_0}{M_{Pl}}}. \qquad (50)$$

Requiring $V_1(0) = V_1(+\infty)$ and V0 = $-\chi\varphi_0/4$, we find the constraint |α| = ln 2. Further, plugging in this value in the expression of α given by Eq. (50), we obtain $\frac{\varphi_0}{M_{Pl}} = \sqrt{3/2}\ln 2 \approx 0.85$.

So, it is possible to heuristically construct the Starobinsky potential, once we considered a quasi-quintessence field. The regime of inflation appears naturally once we violate the no-go theorem.

6 Final outlooks and perspectives

We have highlighted the key characteristics of quasi-quintessence, a field description derived from a Lagrangian formulation that includes a Lagrange multiplier. The quasi-quintessence model is particularly intriguing due to its property of having a vanishing sound speed

while maintaining a generally non-zero equation of state. Consequently, we delved into the concept of a matter-like substance with pressure linked to quasi-quintessence and examined some specific scenarios that arise when the model's free parameters are fixed. In our exploration, we encountered cases resembling radiation and the cosmological constant, and we compared these findings with the more established quintessence framework.

In light of our investigation, we contemplated potential applications for quasi-quintessence and why this fluid can be considered as an effective representation of dark energy, in analogy to models of unified dark energy scenarios [13-25]

To address these questions, we revisited Weinberg's no-go theorem within the context of the cosmological constant problem. We emphasized that solving this problem often necessitates careful fine-tuning of the Lagrangian, effectively shifting the problem rather than resolving it. Leveraging the insights from the no-go theorem, we propose that quasi-quintessence can function as a fluid with a constant generalized kinetic component. This implies a specific transition period during which the no-go theorem is violated. Violation of this theorem suggests that the potential governing the field becomes field-dependent during the transition, a phenomenon we interpret as inflation. In essence, inflation becomes a manifest occurrence that transpires because the no-go theorem is breached for a brief period, precisely the number of e-foldings required to inflate the universe.

Moreover, we derived a plausible effective potential during this transition and a post-transition Hubble rate that bears resemblance to the ΛCDM model. While the standard model is achieved by incorporating an effective cosmological constant contribution, specifically the bare cosmological constant, during the transition phase, the effective potential exhibits characteristics akin to the Starobinsky model, underscoring the validity of our approach [26-28]. We stress that the use of quasi-quintessence to amend the cosmological constant problem is consequence of the cancellation mechanism that we here developed. The latter, in fact, predicts the existence of quasi-quintessence meanwhile it acts to heal the cosmological constant problem.

Summing up, the cancellation mechanism provided in the pioneering work [9] predicts that to cancel the degrees of freedom of the cosmological constant one can invoke the existence of quasi-quintessence. However, an inflationary stage is expected to occur because during the transition the no-go theorem is violated, i.e., the shift symmetry is no longer valid.

Our future research endeavors will focus on elucidating the remaining intricacies of this puzzle. Initially, we aim to provide a more comprehensive interpretation of the results in the context of the no-go theorem, elucidating the conditions under which it becomes possible to physically contravene the theorem. One avenue worth exploring may involve requiring a varying cosmological constant during the transition or, more likely, to violate the shift symmetry and to expect inflation not to lie on this hypothesis, as commonly, instead, it is assumed in current literature. In other words, is that possible to invoke an inflation without shift symmetry? To answer this question, we subsequently intend to offer a more detailed explanation of how particle production genuinely occurs throughout this process, avoiding any reliance on arbitrary fine-tuning. The self-consistency of the model can only be achieved if we refrain from imposing predetermined values to match observations with our theoretical framework.

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