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Phase and group velocities of waves in medium of Quark-gluon plasma

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Wave processes propagating in a medium of collisional and viscous quark-gluon plasma (QGP) are comprehensively analyzed in the paper. To investigate various optical properties of the medium, the longitudinal and transverse dielectric functions of the quark-gluon plasma are taken to study. The calculations of perturbations in these dielectric functions facilitate the derivation of dispersion relations for wave propagation within such media. Through extensive further analysis, the phase and group velocities were calculated for the considered models. The resulting graphs for phase and group velocity distinctly illustrate the dissipative properties inherent in the medium, revealing how these properties influence the propagation speed of both the wave phase and the wave packet of environmental disturbances. This article provides an in-depth analysis of the obtained results, discussing the possible behavior of waves across different models of quark-gluon plasma media. The study concludes by highlighting the achievements and contributions of this research, emphasizing its importance in advancing the understanding of wave dynamics in such extreme states of matter.

Key words: quark-gluon plasma, dielectric function, phase and group velocity, waves in plasma.
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1 Introduction

The quark-gluon plasma (QGP) is a special state of matter under extreme conditions, which is formed at high temperatures and energy densities [1]. Such conditions occur naturally in the cores of massive celestial bodies [2], as well as in the early stages of the development of the Universe [3]. Moreover, such a state of matter under controlled parameters can be obtained in particle collision experiments at ultra-relativistic velocities. Such experiments are carried out at the SPS (Super Proton Synchrotron) [4], LHC (Large Hadron Collider) and RHIC [5] (Relativistic Heavy-Ion Collider).

At high energy densities, hadronic matter ceases to exist and decays into its constituent quarks and gluons. Moreover, quarks and gluons are in unbound states, which is not observed in the normal state of matter – this is the well-known phenomenon of confinement [6]. To summarize the above, the QGP is a state of matter in which free quarks and gluons are in quasi-neutral and thermodynamic equilibrium state. For a more detailed description of the QGP, the following characteristic parameters of this state of matter can be given. When hadronic matter is compressed to high densities of the order of $\sim 1 fm^{-3}$ and heated to high temperatures exceeding the Hagedorn temperature [7], a transition to the QGP occurs at the pseudocritical temperature $T_c = 156,5 \pm 1,5$ MeV, as shown in lattice QCD simulations [8]. It is believed that such parameters occur naturally in the state of matter in the early Universe, which lasts 10^{-5} seconds after the Big Bang. [9-11]

Thus, it makes sense to study collective effects to determine the basic properties of this new state of matter. Such properties can be various static, dynamic, thermodynamic and optical properties. For this paper, optical properties were chosen to study collective effects. The problem of wave propagation in a medium determines the dispersion properties of the medium, which demonstrates the relationship between wave energy and momentum.

To characterize the medium in simulation, two QGP models are used. The first model is described by the Boltzmann equation [12-14], where the distribution function is the probability of finding fermions with two charge values and two spin values. And the interaction between particles is described by the BGK collision integral [15]. The second model is described by the relativistic hydrodynamic equation, where collisions between particles are not taken into account, but the viscosity of the medium is taken into account as a parameter for the loss of wave energy [16-18].

By defining a model of the medium, it is possible to obtain wave dispersion equations, which are used to determine the phase and group velocities of the wave in the medium. The dispersion relations were obtained in the previous work of the authors of the article [19-21]. Thus, this article is a natural continuation of a larger research topic.

This article consists of the introduction presented in this chapter. The next chapter describes the theoretical foundations of the article and the problem statement. Below are the results of solving the problem, as well as a discussion of the results obtained. In the final chapter, the conclusions of this work are presented.

2 Phase velocity

In the previous chapter, two mathematical models for describing QGP were presented. The first model was usually called the collisional QGP model, since in this description the interaction between particles is described using the frequency of particle collisions. The second model was commonly called the viscous QGP model, since collisions between particles are not taken into account here, and the nonideality parameter of the system is calculated taking into account viscosity. A more detailed description of these models is available in [19-21], where the dielectric function is used to formulate the models as an expression for describing the medium. From the course of classical physics, it is known that the dielectric function describes changes in the external field inside the system, that is, it determines the properties of the system by changing the external field. Thus, by studying the properties of the dielectric function, one can find out the properties of the medium itself. Next, by equating the dielectric function to zero, the perturbations of the system are found that describe the wave dispersion in the QGP medium. Once the dispersion relation is obtained, the phase and group velocities can be calculated. In general, phase and group velocities depend on the energy and momentum of the wave, and the properties of the medium (temperature, density, etc.). Based on this, from the values of phase velocities for various media, one can find out the properties of quarks and gluons.

To calculate these velocities, let's first define them in this part. By analogy with classical physics, the phase velocity of a wave is the speed at which the wave phase moves in space without changing the shape of the wave itself.

The phase velocity in a QGP can be significantly different compared to the phase velocity in other media due to the properties of the quarks and gluons that form this plasma.

The group velocity of a wave determines the velocity of propagation of a wave packet, the peak or front of a group of waves. Group velocity is an important property of the medium, which determines the propagation of energy and information. Therefore, studying the group velocity of waves in a QGP demonstrates how quickly disturbances propagate in this medium.

In this part of the article, we will present analytic expressions for phase and group velocity. To do this, we introduce a general expression for a plane wave:

$$\Psi = \Psi_0 \exp i(\omega t - kx - \phi_0). \tag{1}$$

Here, Ψ_0 – amplitude of oscillations, ω – frequency of collisions, k – wave number, ϕ_0 – initial phase of oscillations. Moreover, $\phi(x,t) = \omega t - kx - \phi_0$ – is phase of wave. It means, it is possible to study the dependence of the change in the phase of wave oscillation over time. By definition, phase velocity is the speed at which a point of constant phase propagates

$$\frac{d\phi}{dt} = 0 = \omega - k\frac{dx}{dt} - 0. \tag{2}$$

$$\frac{dx}{dt} = \frac{\omega}{k} = v_p. \tag{3}$$

Using expression (3) one can determine the phase velocity; for this you need to find the ratio of the oscillation frequency and the wave vector.

3 Group velocity

Since phase velocity is formulated, let's introduce group velocity derivation. In an ideal homogeneous medium without wave attenuation (in vacuum), the phase and group velocities are equal.

However, in a real problem there is always wave attenuation due to the imperfection of the medium. That is, the plane wave equation can be modulated. In the following example, we will consider these modulations, which describe the difference between group and phase velocities. For the oscillation frequency we will write $\omega_0 + d\omega$, and for the wave vector we will write $k_0 + dk$. Next, consider the addition of two waves with a small change in modulation

$$\Psi = \exp i[(\omega_0 - d\omega)t - (k_0 - dk)x] + \\ + \exp i[(\omega_0 + d\omega)t - (k_0 + dk)x]. \quad (4)$$

After simple algebraic operations, expression (4) is reduced to the following form

$$\Psi = \exp i(\omega_0 t - k_0 x)\cos(d\omega\, t + dk\, x). \quad (5)$$

In this expression, the exponential part describes high-frequency oscillations that propagate at the carrier speed or phase velocity $v_p = \omega/k$. And the second term of the multiplication, cosine, describes the modulations, which can be equated to zero and the group velocity of the modulations can be found.

$$d\omega\, t + dk\, x = 0 \rightarrow v_g = \frac{d\omega}{dk} = \frac{x}{t}. \quad (6)$$

From the previous expression it follows that to find the group velocity it is necessary to find the derivative of the oscillation frequency with respect to the wave vector. To summarize this chapter, to find the phase and group velocity, one needs to know the wave vector and oscillation frequency, the values of which are determined due to the dispersion relation of the wave.

4 Results

In this part of the paper the plots of the phase and group velocity are presented. Those results were calculated according to the procedure described in the previous part of the paper [19-21].

Fig-1 shows the phase velocity values v_p over the wave number k of a collisional QGP. The values of the wave number were normalized with the plasma frequency ω_p. Since, natural units are used in high-energy physics, the wave number and the plasma frequency have the dimension of energy, so phase and group velocities as well as wave number are dimensionless quantities. Fig-1-a) presents the absolute values of phase velocity for different collision frequencies, and Fig-1-b) presents the comparison of the different oscillation modes to the HTL approximation mode. This can be formulated in the following as $\Delta\tilde{v}_p(\nu) = \frac{[v_p(\nu) - v_p(HTL)]}{v_p(HTL)}$. This formula says that the difference of various oscillation modes and HTL approximation mode is divided by the HTL approximation mode. This structure of figures will be used for the next three figures as well.

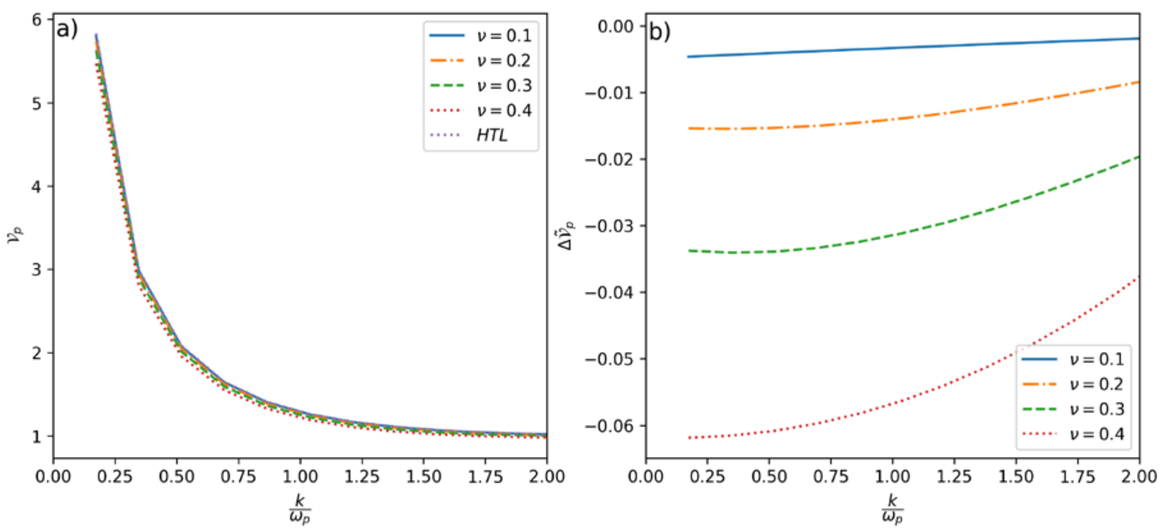

Figure 1 – Phase velocity of collisional QGP: a) values of phase velocity v_p, b) comparison of different modes

In fig-1 and fig-2 collision oscillations were chosen to be small in comparison to oscillation frequency, because propagation waves have to have small damping coefficients.

Fig-2 describes the behavior of group velocity due to wave number in the collisional QGP; the left-hand side plot shows the absolute values and the right-hand side the difference of different oscillating modes. On the other hand, Fig-3 and fig-4 demonstrates the same results for the viscous QGP. For the viscous QGP the values of the free parameter of the model are chosen from the experiments. As it has been written before, the zero value of the viscosity is the same as HTL approximation limit. The value $1/4\pi$ is taken from the theoretical predictions of the smallest viscosity of QGP [22]. The next three is taken from the analysis of results of the experiments, they are listed in accordance with order of the plots. [23-25]

Fig-1-b) shows that the oscillation modes are more different for the small wave number and come closer to each other at higher wave number. One can guess that for even higher wave numbers the different modes will be the same, but the lack of computation power did not allow to calculate further. Moreover, those ranges of wave number are not feasible to analyze in current experimental setups.

Fig-2-b) shows an interesting behavior for the group velocity of collisional QGP. This plot has several points, where different modes have the same group velocity, and they differ more for small wave number and for high wave number.

Fig-3-b) has more predictable behavior with comparison to the fig-2-b). In this plot, phase velocity has the same value for the wave number equals to zero, and the difference grows with increasing wave number. At some point it reaches the extremum point and the difference gets smaller again. And again, one could say that the trend goes to the same point, but the calculation did not support such high values of wave number.

Fig-4-b) presents the comparison of group velocity of viscous QGP, and it shows predictable behavior of difference between various modes. At the small wave number, the difference is the greatest, and it becomes equal at the large wave numbers.

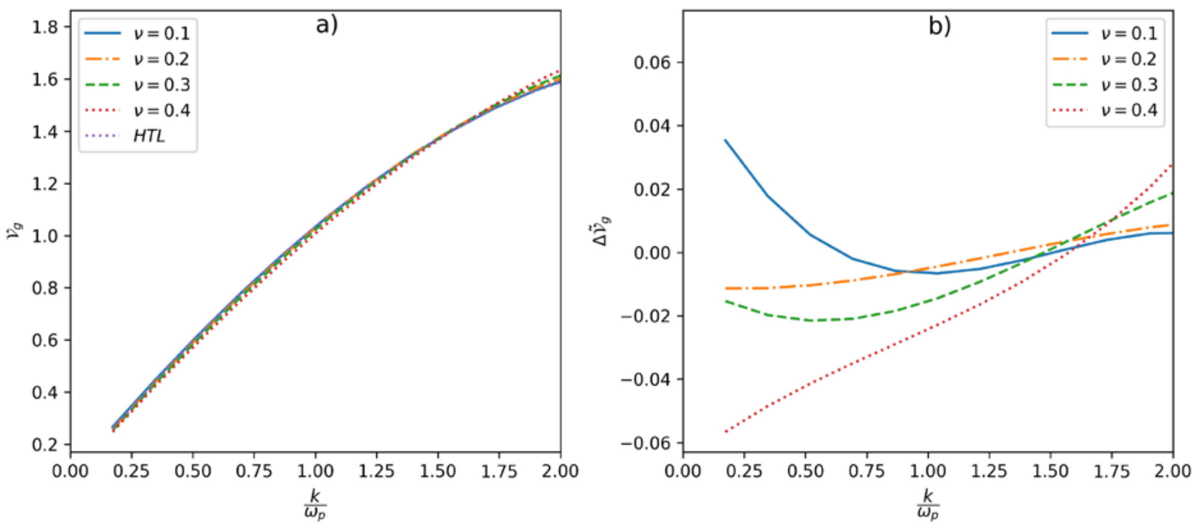

Figure 2 – Group velocity of collisional QGP: a) values of group velocity v_g, b) comparison of different modes

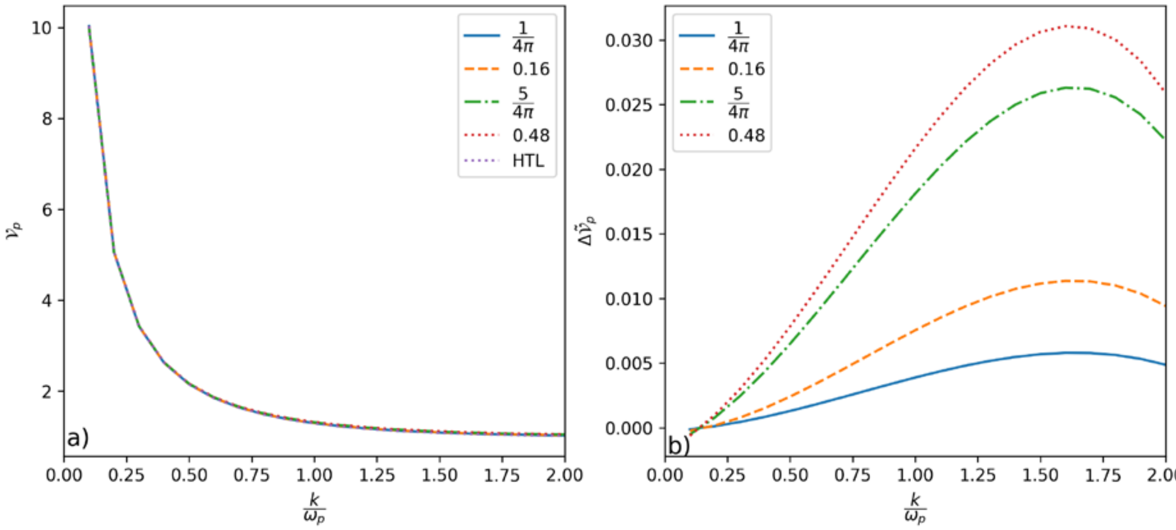

Figure 3 – Phase velocity of viscous QGP: a) values of phase velocity v_p, b) comparison of different modes

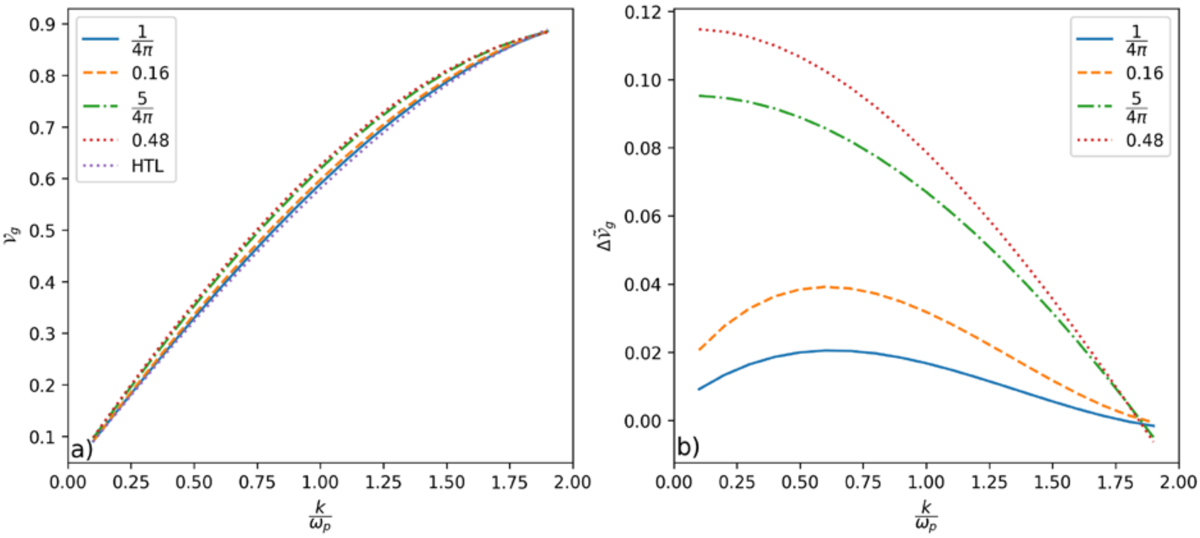

Figure 4 – Group velocity of viscous QGP: a) values of group velocity v_g, b) comparison of different modes

5 Conclusions

The study of phase and group velocities of waves in a quark-gluon plasma medium is an interesting area of research in modern high-energy physics. These parameters may play a key role in understanding the properties of this exotic form of matter and its behavior under extreme conditions of temperature and density. For these reasons we have calculated phase and group velocity for QGP in different models. We have compared these two models with the HTL approximation in order to show the correspondence with other studies of wave propagation in media of QGP. That is why this article can be understood as the part of bigger studies of QGP matter. The chosen mathematical models have limitations in derivation of the dielectric function in order to get analytical results. Those limitations can be found in the original papers and those constraints apply for these results as well. However, the postulated goals of this article can be achieved in the scope of those limitations, because the existence of the wave propagation and the difference of the phase and group velocities in two models were demonstrated. With this, the article concludes with the positive results. The presented results might be used to describe properties of the media of QGP.

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