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Extraction of the radii of ^{9}Be, ^{10}B and ^{11}B nuclei in the approximation of the strongly absorbing nucleus model in elastic scattering reactions of deuterons

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To study the cluster states of light nuclei, in this work, the radii of ^{9}Be, ^{10}B and ^{11}B nuclei were calculated using elastic scattering of deuterons with energies from 11 MeV to 28 MeV. The values of the free parameters from the complex angular momentum method were calculated using fits to the single-valued minimum Pearson values. The fits were carried out by describing the theoretical curve of the experimental data for the first two oscillations of the Fraunhofer-type nuclear diffraction of the differential cross sections of elastic scattering of deuterons. Moreover, the array of experimental data points was not always unambiguous, and this led to difficulties in the quality of fitting free parameters, which affected the results of radius calculations. The results of the radii are presented in the paper, the values of which were compared with the values of the world literature data. The obtained calculated values of the radii of nuclei for ^{9}Be are in satisfactory agreement with the exception of some values. For ^{10}B and ^{11}B, there is a systematic excess of the obtained radii over the literature values. It is possible that such excess is due to the fact that when calculating the radii of the nuclei under study, the structure of the deuterons and their own radius, which increases the radius of interaction, were not taken into account.

Key words: elastic scattering; deuterons; light nuclei; Fraunhofer-type nuclear diffraction; root-mean-square radius of a nucleus.
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1. Introduction

Light nuclei, the structure of which is described by theoretical models of cluster states [1-4], are currently being actively studied. In most cases, such a theoretical approach describes the distribution of nuclear density quite well. This phenomenon, in turn, manifests itself in the form of anomalous sizes of mean-square radii, such as the halo in [5]. Nuclei in the cluster model approach have been studied for quite a long time; as an example, such studies can be traced in works [6-8]. Determining the root-mean-square radius of a nucleus, and if this value goes beyond classical concepts, then this allows us to draw conclusions about further study of the nuclei under study and the possible pronounced cluster configuration of these nuclei. The relevance and importance of studying differential, integral and total cross sections of light incident particles is, first of all, the solution of fundamental problems of nuclear astrophysics and applied problems of thermonuclear energy and radioecology.

In the present work, calculations of the radii of ^{9}Be, ^{10}B and ^{11}B nuclei were performed. The radii of nuclei were calculated using the complex angular momentum method (CAMM). The CAMM was implemented in an analytical form by the authors of [9-11] from a rigorous quantum-mechanical expression for the elastic scattering amplitude expanded in partial waves.

2. Materials and methods

Basic information about the structure and properties of nuclei and the mechanisms of nuclear reactions is obtained primarily from elastic scattering reactions.

Studies on the structure of light nuclei based on the cluster approach, in particular in [12-22], show that the relevance of using the cluster model to describe light nuclei, up to 40-Ca, is very high. This approach allows us to interpret such nuclei as a group of different clusters interacting with each other. In this case, the dynamic association in the ratio of one type of cluster to another from nucleus to nucleus, or even from isotope to isotope, can be radically different. Such a difference and the existence with a greater probability of certain cluster configurations affects the root-mean-square radius, the blurriness of the edge of the nucleus and the nuclear deformation. In turn, such nuclear parameters are extracted from various nuclear reactions. In particular, in this work, the root-mean-square radius is extracted from oscillations of differential cross sections of elastically scattered deuterons. Expansion of the scattering amplitude into the sum of partial scattered waves

$$\frac{d\sigma(\theta)}{d\Omega} = \left|A(\theta)\right|^2 = = \frac{1}{2ik}\cdot\sum_{l=0}^{\infty}[2l+1]\cdot\left[e^{2i\eta_l}-1\right]\cdot P_l\left(\cos(\theta)\right), \quad (1)$$

where η_l – scattering phase

$$\eta_l = \sigma_l + \delta_l . \quad (2)$$

σ_l – Coulomb phase; δ_l – nuclear phase.

Scattering matrix has the form

$$S_l = e^{2i\eta_l} = e^{2i(\sigma_l+\delta_l)}, \quad S_l = 1 - \left[1+\exp\frac{l-l_0}{\lambda}\right]^{-1}. \quad (3)$$

where l is the angular momentum, λ is the diffuseness parameter of the edge of the nucleus. Section (1) taking into account (3) will have the following form

$$\frac{d\sigma(\theta)}{d\Omega} = \sigma_0(\theta) = = \frac{8\pi}{k^2}|a|^2 l_0 \frac{b^2+\cos^2\left((l_0+0,5)\theta+\gamma\right)}{\sin(\theta)\cdot e^{2\beta\theta}} \quad (4)$$

where $|a|, l_0, \beta, b, \gamma$ are the free parameters of the theory. The analytical form (4) with the corresponding restrictions in the paradigm of the strongly absorbing nucleus model is the method of complex angular momenta [9-11].

In elastic scattering, nuclear diffraction of different types can be observed in differential angular distributions depending on the properties of the incident wave from the beam of charged particles interfering with the properties of the target nucleus. The conditions for the occurrence of one or another type of nuclear diffraction are presented below.

1. $kR \sim 1$ – Rutherford scattering;
2. $\begin{cases} kR >> 1 \\ n \sim 1 \end{cases}$ – Fraunhofer type diffraction;
3. $\begin{cases} kR >> 1 \\ n >> 1 \end{cases}$ – Fresnel type diffraction. (5)

where $n = \frac{Z_1 Z_2 e^2}{\hbar \cdot v}$ – Coulomb parameter (Sommerfeld parameter); $k = \frac{\sqrt{2\cdot M\cdot E}}{\hbar}$ – wave number. Fraunhofer diffraction occurs when the wavelength of the incident particle is less than the radius of the nucleus. This resembles the passage of light through a narrow slit, where a characteristic interference pattern with alternating light and dark bands is observed on the screen. In the case of nuclear diffraction, these bands correspond to different scattering angles of the particle. For this process to occur, it is important that the Sommerfeld parameter is less than 1. This parameter characterizes the influence of the Coulomb field of the nucleus on the trajectory of the particle. The smaller the Sommerfeld parameter, the less the influence of the Coulomb field, and the clearer the diffraction pattern. Fresnel diffraction differs from Fraunhofer diffraction in that the Sommerfeld parameter in this case is greater than 1. This means that the Coulomb field of the nucleus strongly influences the motion of the particle. As a result, we observe interference between nuclear scattering (reflection from the nucleus) and scattering in the Coulomb field. This interaction leads to more complex diffraction patterns. Let's consider each type in more detail.

Fraunhofer diffraction: Imagine a beam of electrons directed at an atomic nucleus. This makes Fresnel diffraction more difficult to analyze, but also more informative in terms of studying the structure of the nucleus.

This paper presents a literature experimental analysis of differential angular cross sections of elastic scattering of deuterons under Fraunhofer-type nuclear diffraction conditions.

The search for the optimal parameters of the CAMM (4) was carried out by minimizing the value χ2

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(\sigma_i)_{teor} - (\sigma_i)_{\exp}}{\Delta(\sigma_i)_{\exp}}\right]^2, \qquad (6)$$

where $(\sigma_i)_{teor}$ and $(\sigma_i)_{\exp}$ – theoretical and experimental cross sections, N – number of measured points.

Figure 1 shows the picture of the values of the χ^2 isolines of the free parameters of the model (4) calculated for the reaction 11-B(d,d)11-B Ed=18 MeV. The search for optimal parameters was proposed in [23]. In this work, the parameters of the theory were adjusted to the experiment using this method.

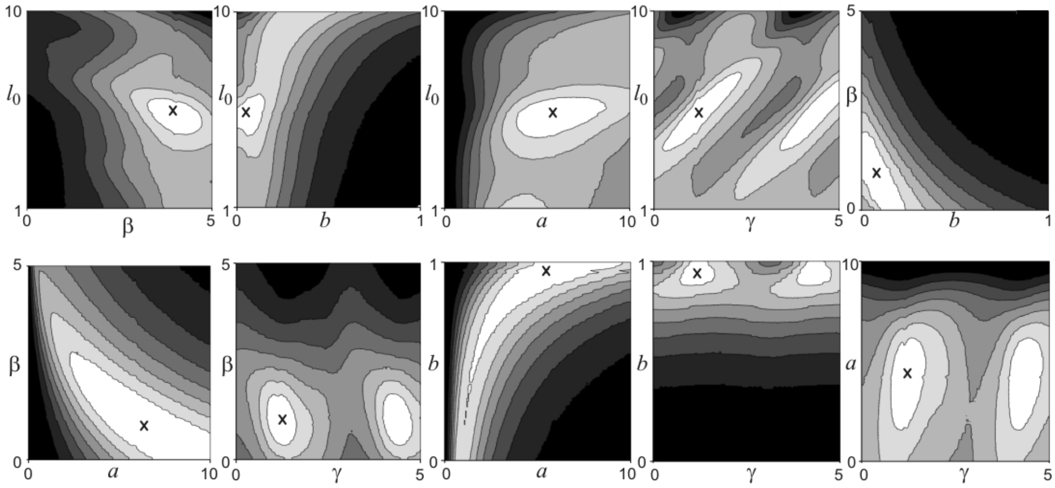


Figure 1 - χ2- distributions for five pairs of free parameters of the method of complex angular momenta of elastically scattered deuterons on ^{11}B, E_d=18 MeV

Based on the found free parameters of the CAMM (4), the radius of interaction of the incident charged particles with the studied nuclei was calculated

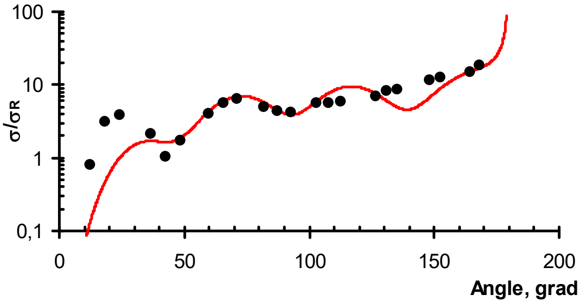
$$R_{\text{int}} = \frac{1}{k}\left[n + \sqrt{n^2 + l_0(l_0+1)}\right], \qquad (7)$$

where k – wave number; n – Coulomb parameter, l_0 – orbital momentum from (4).

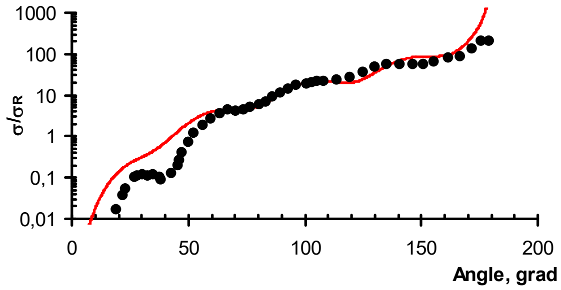
3. Results and discussion

In this work, using our own developed software for searching for optimal values, free parameters of the model (4) were found.

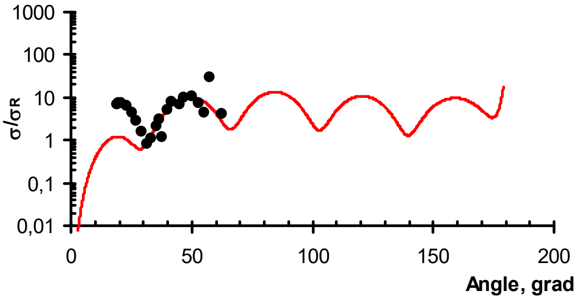
Figures 2-5 show the dependences of the angular distributions of differential cross sections of elastically scattered deuterons on ^{9}Be, ^{10}B and ^{11}B with energies from 11 MeV to 27.7 MeV. The points in the angular dependences are experimental data taken from the international nuclear database NNDC. The criteria for finding the optimal free parameters were the following considerations. In the region of small angles, where Coulomb forces compete and in some cases prevail over nuclear forces, this method, the method of complex angular moments, describes the experimental data poorly. This is explained by the fact that the Coulomb component does not enter into the section (4) itself. At angles greater than 90 degrees, inelastic nuclear absorption begins to enter, as well as exchange processes that are not taken into account in the scattering matrix. Thus, in this model (method) the region of angles is investigated, in which nuclear scattering mechanisms prevail. The CAMM allows to reveal these regions, in which if the theory describes experimental data well, then strong discrepancies are observed beyond their boundaries. Such discrepancies are manifested in the discrepancy between the oscillation frequency, phase, slope of the envelope and the absolute value of the cross section. Table 1 presents the values of the free parameters of the model and the calculated interaction radii.



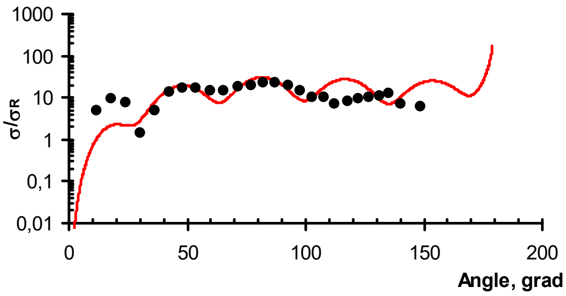
E_d=11 MeV (points – [24], curve – CAMM (4))



E_d=13.6 MeV (points – [25], curve – CAMM (4))

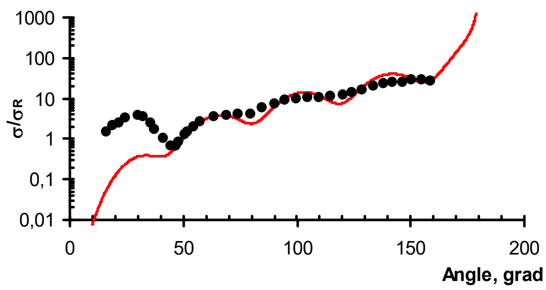


E_d=18 MeV (points – [26], curve – CAMM (4))



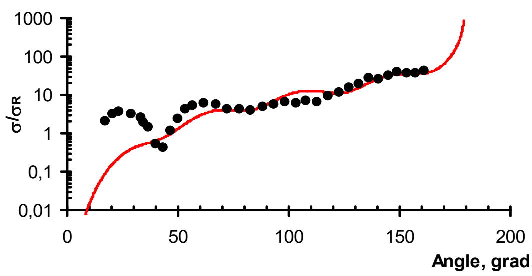
E_d=27.7 MeV (points – [27], curve – CAMM (4))

Figure 2 - Differential cross sections of elastically scattered deuterons on ^{9}Be



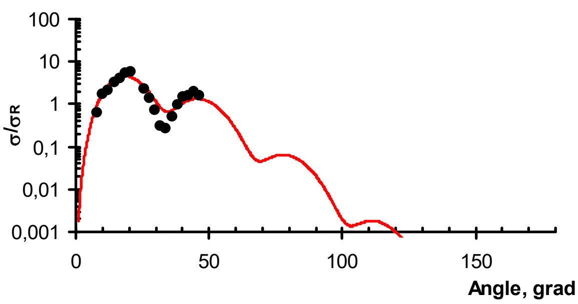
E_d=11.8 MeV (points – [28], curve – CAMM (4))

Figure 3 - Differential cross sections of elastically scattered deuterons on ^{10}B

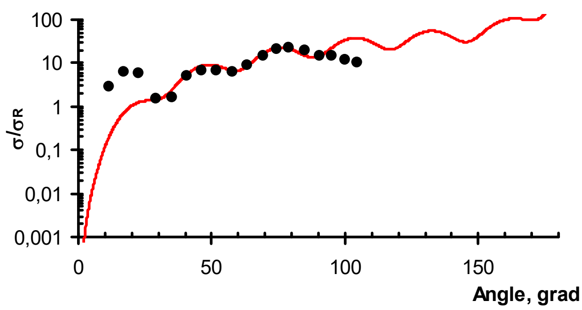


E_d=11.8 MeV (points – [28], curve – CAMM (4))

Figure 4 - Differential cross sections of elastically scattered deuterons on ^{11}B



E_d=18 MeV (points – [26], curve – CAMM (4))



E_d=27.7 MeV (points – [27], curve – CAMM (4))

Figure 5 - Differential cross sections of elastically scattered deuterons on ^{11}B

Table 1 - Values of free parameters of the CAMM (4)

	E, MeV	k, 1/fm	n	R_{int}, fm	l_0	β	b	|a|	γ
^{9}Be(d,d)	11.00	0.84	0.24	4.91	3.40	0.80	0.90	0.70	1.50
	13.60	0.93	0.22	4.71	3.70	0.00	1.10	0.20	2.10
	18.00	1.07	0.19	4.72	4.40	1.10	0.40	1.20	2.20
	27.00	1.33	0.15	3.88	4.55	1.00	0.65	1.17	2.25
^{10}B(d,d)	11.80	0.89	0.30	5.54	4.20	0.10	0.70	0.30	1.20
^{11}B(d,d)	11.80	0.90	0.30	5.21	3.90	0.22	1.20	0.27	1.40
	18.00	1.11	0.24	4.97	4.80	4.00	0.50	5.80	1.60
	27.70	1.38	0.20	4.70	5.80	0.50	0.90	0.50	1.25

Thus, in Figure 2, during the interaction of deuterons with an energy of 11 MeV with ^{9}Be, one maximum of Fraunhofer diffraction is well described, which stretched over the angular range of scattered deuterons from 50 to 100 degrees. The first maximum is not described due to the absence of Coulomb interaction in this model. At high energies, 13.6 MeV, 27.7 MeV, two maxima are described quite well. In the experimental data at the deuteron energy of 18 MeV, unfortunately, the authors of [26] did not obtain enough data. For ^{10}B and ^{11}B, similar trends were observed in the fitting of the free parameters of the model. However, when describing the angular distribution of elastically scattered 18 MeV deuterons on ^{11}B, the envelope slope should most likely be different. At least not as different from the envelope slope as shown in the angular distributions of elastically scattered 27.7 MeV deuterons. Nevertheless, the description of the first maximum fits well into the overall picture of the interaction radii at different energies, as shown in Figure 6. Nevertheless, for one or two Fraunhofer-type oscillations, it is possible to select the parameter l_0, which is associated with the interaction radius (7).

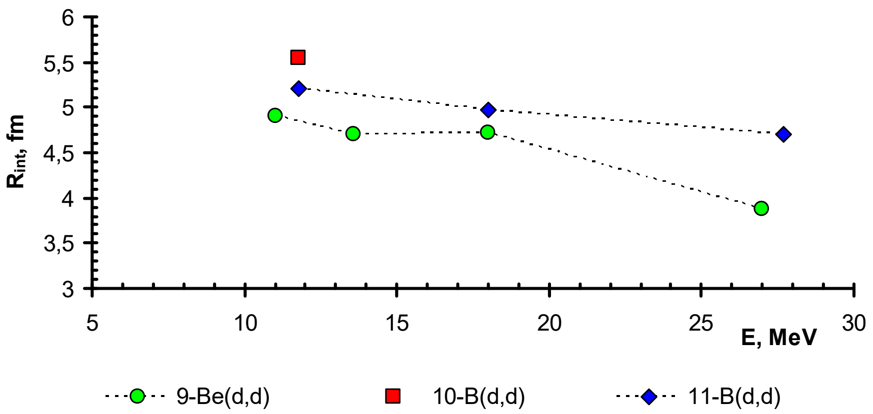


Figure 6 - Radii of interaction of deuterons with nuclei ^{9}Be, ^{10}B, ^{11}B

To determine the radii of the nuclei under study, it is necessary to remove the energy dependence. As was shown in [21], the radius of the nucleus can be determined as

$$R = R_{\text{int}} - \lambda_d - r_{NN}, \quad (8)$$

where $\lambda_d = 1/k$ – de Broglie wavelength of deuterons, $r_{NN} = 1,0\ Fm$ – nuclear force range. Figure 7 shows the radii of the studied nuclei, calculated using this CAMM method (dots), in comparison with literature data (straight lines) [29, 30].

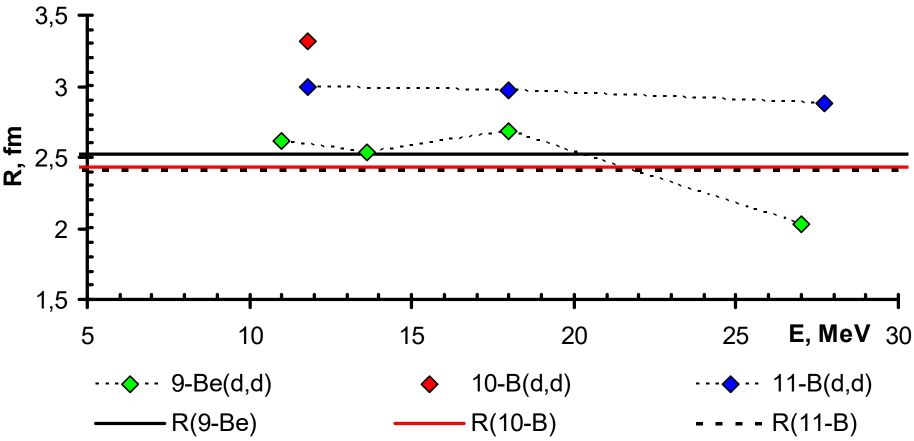


Figure 7 - Radii of nuclei ^{9}Be, ^{10}B, ^{11}B

At energies up to 20 MeV, the calculated radii for ^{9}Be agree satisfactorily with the literature data. However, at a deuteron energy of 27.7 MeV, the radius value is significantly smaller. Perhaps, it is necessary to conduct a more accurate analysis and study the reactions of elastic scattering of deuterons close to this energy, or this can be explained by the fact that at a given deuteron energy, the de Broglie wavelength is approximately equal to half the radius of the alpha particle; scattering may occur on a combination of nucleon associations representing alpha clusters in ^{9}Be. For boron isotopes, the radius values are more. Perhaps, this is due to the internal structure of the deuteron, its large radius and, in this regard, the calculation of the radii by (8) will not be accurate, where the incoming components do not take into account the properties of deuterons.

4. Conclusions

Thus, as a result of calculations to extract the optimal free parameters of the complex angular momentum method, the interaction radii of deuterons with energies from 11 MeV to 28 MeV on ^{9}Be, ^{10}B and ^{11}B nuclei were calculated. The optimal parameters were extracted from the differential cross sections of elastically scattered deuterons on the nuclei under study with energies in the above-mentioned ranges, at which Fraunhofer-type nuclear diffraction oscillations were clearly evident. The radii were calculated using the optimal free parameters of the CAMM. However, as a comparison with the literature values of the radii of the nuclei under study showed, there is a clear excess of the obtained values. For such an explanation, it is necessary to conduct an analysis on a large array of experimental data on elastic scattering of light ions, with good angular resolution and a wide angular range.

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