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Cosmological model in modified *f(R, G)* Gauss-Bonnet gravity

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In this work, the Gauss-Bonnet model of modified gravity is investigated, where some arbitrary function *G* is added to the Einstein-Hilbert action. This theory explains the accelerated expansion of the Universe. In this work. The article proposes *F(R, G) = f(R) + η(G)* modified gravity, which considers two gravities *f(R)* and *η(G)*, where *f(R)* is a function from the Ricci scalar, *η(G)* is a function from the Gauss-Bonnet invariant. The model is considered in a flat, isotropic and homogeneous Universe. As a result of some mathematical formalism, the dependence of the function *f(R)* on the scalar of curvature *R* and on *t* time is found. Geometric and dynamic parameters of the cosmological model *F(R, G) = f(R) + η(G)* were analyzed. Equations of motion and cosmological parameters, such as the Hubble parameter and scale factor, were obtained for the investigated model. Analyzing the obtained solutions of the scale factor, it was shown that the model describes the exponential acceleration of the Universe. Thus, it was found that the cosmological model under study has a similar interpretation to the de Sitter cosmological model.

Key words: Gauss-Bonnet gravity, *f(R, G)* gravity, Hubble parameter, acceleration of the Universe.
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1. Introduction

Many astronomical studies have shown that the Universe is currently expanding at an accelerated rate [1]-[5]. Most cases of general relativity (GR) are generalized by incorporating scalar curvatures, higher-order curvature terms, and also connections with dynamic scalar fields [6], [7]. Consequently, there is growing interest in studying modifications and generalizations of Einstein's theory. Various approaches and models exist for investigating the expansion of the Universe. Effective cosmological results can be obtained using modified theories of gravity. Various modified models and gravitational theories have been proposed, including *f(R)* gravity [8]-[10], $f(G)$ gravity [11] , scalar-tensor theory [12], $f(R,T)$ gravity [13] and $f(R,G)$ gravity [11], where R is the Ricci scalar, G is the Gauss-Bonnet invariant, T is the torsion.

f(R) gravity, a kind of modified theory of gravity that generalizes Einstein's general theory of relativity. Over the past few decades, various forms of the $f(R)$ function have been investigated. Among these functions there are quite viable ones that correctly describe cosmological dynamics, a smooth transition between different cosmological epochs [14]. The cosmological interest in $f(R)$ gravity arises from the fact that these theories naturally demonstrate the late-time accelerated expansion of the Universe without the need for matter fields like dark energy. In a study by [15], a scheme for cosmological reconstruction of $f(R)$ gravity is presented. Among other existing theories, it can be shown that gravitational models based on $f(R)$ describe the transition from a matter-dominated phase to an accelerated phase [16]. However, it is well-known that $f(R)$ gravity has some imperfections. For instance, at the nonlinear level, issues related to curvature singularities arise [17]. As a result of the classical GRT tests obtained, most of the proposed $f(R)$ models are excluded in the limitations of the Solar System regime. In order to circumvent these imperfections, gravity $f(R)$ has been expanded to take into account additional scalars in the Einstein-Hilbert action. In this regard, an optimistic alternative arises, such as $f(R,G)$ gravity [18-24]. The stability of cosmological solutions in $f(R,G)$ gravity is discussed in [25]. Theories like $f(R,G)$ satisfy the constraints of the Solar System [26].

In the Gauss-Bonnet theory of gravitation the Einstein action is modified by the function $f(G)$, where an arbitrary function G is a quadratic invariant of the Gauss-Bonnet equation [11]. It is known that G is a topological invariant in four dimensions, which participates in the formulation of quantum field theory in curved space. The invariant G arises under gravitational influences containing second-order curvature invariants. The Gauss-Bonnet function $f(G)$ is added to the gravitational interaction to explain the accelerating expansion of the Universe at late times [27]. Moreover, such modified Gauss-Bonnet gravity can describe the transition from deceleration to acceleration as well as the phantom gap crossing. One can search for more serious restrictions on its form by comparing the theory with observational data. Models containing the Gauss-Bonnet invariant have attracted interest because of the ability of G to simplify the dynamics of the system. In recent years, modified theories associated with the topological Gauss-Bonnet term have been studied in depth [28]. In [29], a class of Horndeski Lagrangian, with a scalar k-essence field associated to the Gauss-Bonnet term, is considered. A reconstruction method is proposed to derive viable models in accordance with cosmological data. The Gauss-Bonnet invariant is also considered in the Λ CDM cosmological model [30]. It is shown that the Λ CDM model can be explained in such theories, where the problem of the cosmological constant is explained in the form of a modified of the cosmological constant.

In this paper, the evolution of the Universe is investigated by considering two gravity separated functions, gravity $f(R)$ and $\eta(G)$, where $f(R)$ is a function of the Ricci scalar R and $\eta(G)$ is a function of the Gauss-Bonnet invariant G [31]-[35]. This theory without any cosmological constant can predict different phases of the evolution of the universe [36], [37]. The G in the curvature invariant corresponds to the coevolution of the early Universe. Moreover, this theory describes accelerating waves of celestial objects. It also effectively explains the transition from the deceleration phase to the acceleration phase [38]. Thus, it is possible to construct feasible and consistent modified models using $f(G)$ [11], [39]. Section 2 presents the mathematical formalism of the cosmological model $F(R,G)$ of gravity. The equations of motion were derived and solutions of the unknown functions $f(R)$, $\eta(G)$ and the Hubble parameter, scale factor, were shown. In Sec. 3, cosmological parameters such as pressure, energy density, and state parameter are found and their graphical behavior is shown.

2. The cosmological model of $F(R,G)$ gravity

Consider the following action for $F(R,G)$ gravity

$$S = \int \sqrt{-g}\left[\frac{1}{2k^2}F(R,G) + L_m\right]d^4x, \quad (1)$$

where g is the metric determinant, L_m – standard matter Lagrangian, $k^2 = 8\pi G_N$, G_N is the Newtonian gravitational constant and the speed of light c is assumed to be 1. Now it is necessary to bring Lagrangian into canonical form $L(a,\dot{a},R,\dot{R},G,\dot{G},t)$ from the action (1) to obtain the equation of motion. Here $a = a(t)$ is the scale factor, dependent on cosmological time t and defined in the Friedman-Roberston-Walker metric (FRW)

$$ds^2 = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2), \quad (2)$$

Next, using the Lagrange multiplier method (see for example [40]), we can set R and G as constraints on the dynamics. To eliminate high-order derivatives, we select a suitable Lagrange multiplier and integrate by parts. We rewrite action (1) for flat FRW metric as follows:

$$S = \int d^4xa^3\left[f(R) + \eta(G) - \alpha\left(R - 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\right) - \beta\left(G - \frac{24\dot{a}^2\ddot{a}}{a^3}\right)\right], \quad (3)$$

here the Ricci scalar R and the Gauss-Bonnet invariant G are defined as follows in terms of the Hubble parameter $H=\frac{\dot{a}}{a}$ for FRW metric:

$$R=6\dot{H}+12H^2, \tag{4}$$

$$G=24H^2(\dot{H}+H^2). \tag{5}$$

At (4) α and β are the Lagrangian multipliers that can be directly found by varying with respect to R and G, giving $\alpha=f_R,\beta=\eta_G$, respectively. Where the indices denote derivatives with respect to the given variables $f_R=\frac{df(R)}{dR},\eta_G=\frac{d\eta(G)}{dG}$.

The equation of action (4) is reduced to the following form

$$S=\int d^4x\left[a^3f+a^3\eta^3-a^3Rf_R+6a^2\ddot{a}f_R+6a\dot{a}^2f_R-a^3G\eta_G+24\dot{a}^2\ddot{a}\eta_G\right] \tag{6}$$

According to the equation of action (4), we write the Lagrange function as

$$L=a^3f+a^3\eta-a^3Rf_R-6a^2\dot{a}\dot{R}f_{RR}- \\ -6a\dot{a}^2f_R-a^3G\eta_G-8\dot{a}^3\dot{G}\eta_{GG}. \tag{7}$$

The Euler-Lagrange equation is written in the following form

$$\frac{\partial L}{\partial q}-\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)=0, \tag{8}$$

Also from energy condition

$$E_L=\left(\frac{\partial L}{\partial \dot{q}}\right)\dot{q}-L=0, \tag{9}$$

where $q=a,R,G$ are generalized variables.

For the FRW metric the pressure p and the density ρ are defined as follows

$$p=-(3H^2+2\dot{H}), \tag{10}$$

$$\rho=3H^2. \tag{11}$$

Using equation (5) and (7) we find

$$p=\frac{1}{2f_R}\left[f+\eta-Rf_R-G\eta_G+4H\dot{R}f_{RR}+(2\dot{R}f_{RR})_t+8H^2(\dot{G}\eta_{GG})_t+16H\dot{G}\eta_{GG}(\dot{H}+H^2)\right], \tag{12}$$

$$\rho=3H^2=\frac{1}{2f_R}\left[-f-\eta+Rf_R+G\eta_G-6H\dot{R}f_{RR}-24H^3\dot{G}\eta_{GG}\right] \tag{13}$$

Adding up equations (13) and (14), we obtain the following equation

$$\dot{H}=\frac{1}{2f_R}\left[H\left(\dot{R}f_{RR}+4H^2\dot{G}\eta_{GG}\right)-\left(\dot{R}^2f_{RRR}+4H^2\dot{G}^2\eta_{GG}\right)-\left(\ddot{R}f_{RR}+4H^2\ddot{G}\eta_{GG}\right)-8H\dot{H}\dot{G}\eta_{GG}\right] \tag{14}$$

Let's denote

$$A=\dot{R}f_{RR}+4H^2\dot{G}\eta_{GG}, \tag{15}$$

then

$$\dot{A}=\left(\dot{R}^2f_{RRR}+4H^2\dot{G}^2\eta_{GG}\right)+ \\ +(\ddot{R}f_{RR}+4H^2\ddot{G}\eta_{GG})+8H\dot{H}\dot{G}\eta_{GG}. \tag{16}$$

Using (18) and (19) equations, we obtain equation (17) in a simplified form

$$\dot{H} = \frac{1}{2f_R}\left(HA - \dot{A}\right), \tag{17}$$

To solve (20) the differential equation, consider the following case

$$f_R = \frac{A}{2}. \tag{18}$$

Then equation (18) is reduced to a differential equation with separated variables

$$\dot{H} - H = -\frac{\dot{A}}{A} = C_1, \tag{19}$$

where $C_1 = const$.

The solution of the differential equation (22) can be found in the following form

$$H = e^{(t-t_0)} - C_1, \tag{20}$$

where t_0 - present current time and t variable time, those $t_0 > t$,

$$A = e^{C_1(t-t_0)}. \tag{21}$$

According to (18) and (21) f_R is written as

$$f_R = \frac{e^{-C_1(t-t_0)}}{2}. \tag{22}$$

Considering that $\left(f_R\right)_t = \dot{R}f_{RR}$ and $(\eta_G)_t = \dot{G}\eta_{GG}$, where t in the index means the time derivative, then the equation (15) can be written in the following form

$$\frac{2f_R - (f_R)_t}{4H^2} = (\eta_G)_t. \tag{23}$$

Integrating equation (23), we obtain the following

$$\eta_G = -\int \frac{C_1 e^{-C_1(t-t_0)}}{8(e^{t-t_0} - C_1)^2}dt. \tag{24}$$

Next, consider the value of the equation for f_R. Substituting (20) into (2) we get the quadratic equation

$$12\left[\left(2f_R\right)^{-1/C_1} - C_1\right]^2 + 6\left(2f_R\right)^{-1/C_1} - R = 0. \tag{25}$$

Denoting $b = \left(2f_R\right)^{-1/C_1}$. we get the quadratic equation

$$b^2 - \left(2C_1 - \frac{1}{2}\right)b + (C_1)^2 - \frac{R}{12} = 0, \tag{26}$$

Solving this equation we get

$$b_{1,2} = 2\left(C_1 - \frac{1}{4}\right) \pm \sqrt{\frac{R}{3} - 2C_1 + \frac{1}{4}}. \tag{27}$$

Considering (22) and (27) we write the equation for f_R

$$f_R = \frac{1}{2}\left[C_1 - \frac{1}{4} \pm \frac{1}{2}\sqrt{\frac{R}{3} - 2C_1 + \frac{1}{4}}\right]^{-C_1}. \tag{28}$$

Since the solution of (24) and (28) equations is complex, consider a special case for $C_1 = -1$ and obtain solutions of integral equations in the following form

$$\eta_G = -\frac{1}{8\left(e^{(t-t_0)} + 1\right)} \tag{29}$$

and

$$f(R) = -\frac{5}{8} + \frac{1}{24}\sqrt{12R + 81}. \tag{30}$$

The derivatives of functions $\eta(G)$ and $f(R)$ on time t are defined as

$$\dot{\eta} = \eta_G \dot{G}, \tag{31}$$

$$\dot{f} = f_R \dot{R}. \tag{32}$$

Then considering (20), we write the derivative of the function G and R in (4), (5) by time t

$$\dot{G} = 48\left(e^{2(t-t_0)} + e^{(t-t_0)}\right) \times$$

$$\times \left(e^{(t-t_0)} + \left(e^{(t-t_0)} + 1\right)^2\right) +$$

$$+24\left(e^{(t-t_0)} + 1\right)^2\left(3e^{(t-t_0)} + 2e^{2(t-t_0)}\right) \quad (33)$$

$$\dot{R} = 24e^{2(t-t_0)} + 30e^{(t-t_0)}. \quad (34)$$

Substituting equations (29) and (33) into (31) we obtain the function η dependent on t

$$\eta(t) = -4e^{3(t-t_0)} - \frac{33}{2}e^{2(t-t_0)} - 15e^{(t-t_0)}. \quad (35)$$

Using equations (22) and (34) we obtain the function f dependent on t

$$f(t) = 4e^{3(t-t_0)} + \frac{15}{2}e^{2(t-t_0)}. \quad (36)$$

3. Cosmological parameters

Cosmological parameters, global parameters of the Universe that characterize its composition and dynamics, are determined according to observational data or derived from them. The main cosmological parameters considered in this paper are the Hubble parameter, the scale factor, and the equation of state parameter relating its pressure and density. The most accurate measurements of the global parameters of the Universe are obtained from observed data on supernovae of type Ia stars and from the anisotropy characteristics of the relic radiation. In addition, data from the cosmic distance scale are used to measure the Hubble parameter.

Using equations (18) we can find the scale factor a in the following form

$$a = e^{\exp(t-t_0) - C_1(t-t_0) + C_2}. \quad (37)$$

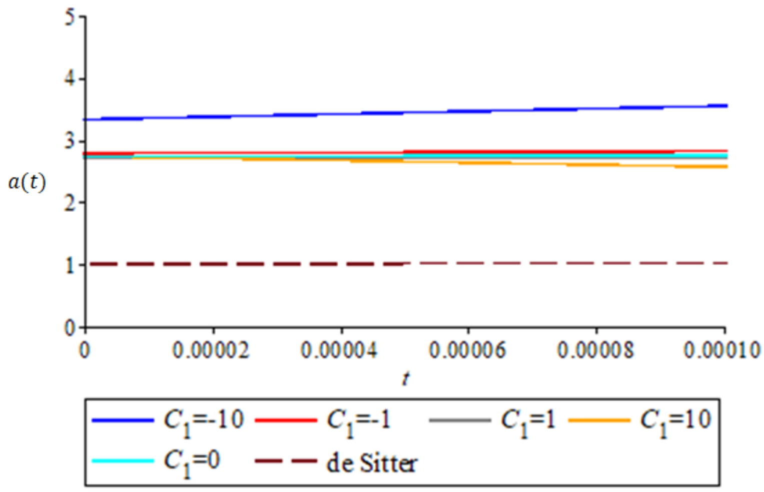

Figure 1 – Variation of scale factor a over time t at $C_1 = -10$ (blue line), $C_1 = -1$ (red line), $C_1 = 0$ (blue line), $C_1 = 1$ (gray line), $C_1 = 10$ (yellow line), de Sitter model (dashed line)

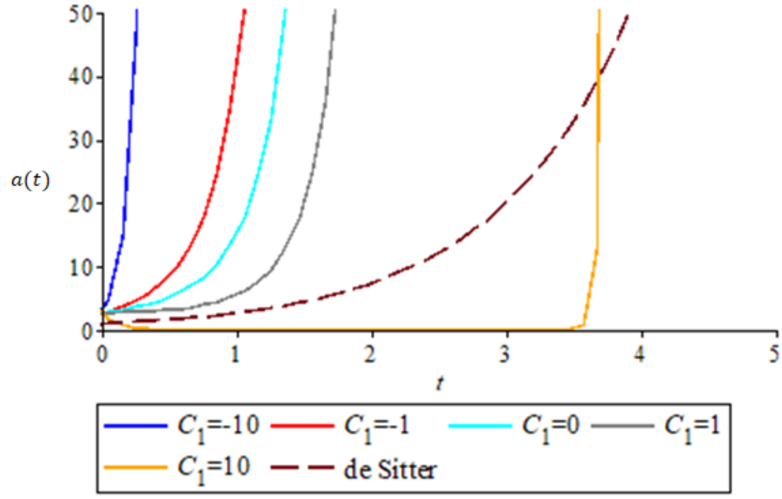

Figure 2 – Variation of scale factor a over time t at $C_1 = -10$ (blue line), $C_1 = -1$ (red line), $C_1 = 0$ (blue line), $C_1 = 1$ (gray line), $C_1 = 10$ (yellow line), de Sitter model (dashed line)

The cosmographic evolution of the scale factor can be seen in Figure 1, at $C_2 = 0$. In Figure 1 we can observe the evolution of the scale factor at early times for the cosmological model under study. We also compare the behavior of these graphs with the behavior of the de Sitter model graph, since the de Sitter model also describes the exponential expansion of the Universe. Epochs from the Big Bang 10^{-43} s to the quark confinement 10^{-4} s, including the 10^{-36} s inflation stage, are considered. The graph showed that in this time interval all values are stationary and have larger values of the scale factor relative to the Einstein Universe. At large negative values of C_1 the values of the scale factor are larger.

Figure 2 shows the variation of the plots at later times. As can be seen, the scale factor of the Gauss-Bonnet model at $C_1 = -10;-1;0;1$ grows exponentially faster compared to the de Sitter model. Figure 2 shows that at large positive values of C_1 the stationary period is longer than at smaller and negative values.

The expansion of the Universe is classified using different phases of the ω state parameter. The state parameter ω is an immeasurable quantity and defined as

$$\omega = \frac{p}{\rho}, \tag{38}$$

where p is the pressure and ρ is the energy density of the matter distribution, equal to the ratio of the total average density of the Universe to the critical density.

p and ρ can be written in the following form

$$p = -3e^{2^{2(t-t_0)}} - 8e^{t-t_0} + \frac{3}{e^{t-t_0}} - 3 \tag{39}$$

and

$$\rho = 3e^{2(t-t_0)} + 6e^{t-t_0} - \frac{3}{e^{t-t_0}} + 3. \tag{40}$$

Substituting (39), (40) into the equation of state (38), we obtain ω

$$\omega = -1 - \frac{2e^{t-t_0}}{3\left(e^{t-t_0}+1\right)^2 - \frac{3}{e^{t-t_0}}}. \tag{41}$$

The graphical result of p and ρ can be seen in the figures below

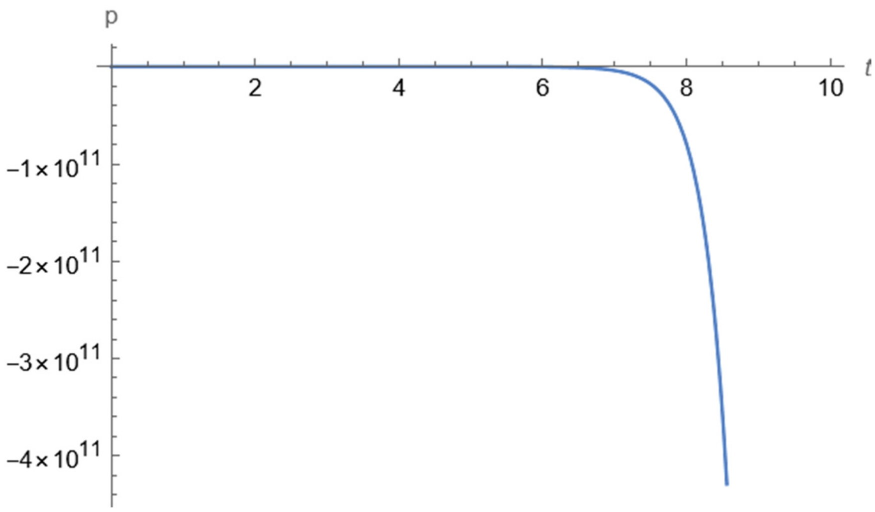

Figure 3 – Change of pressure p over time t

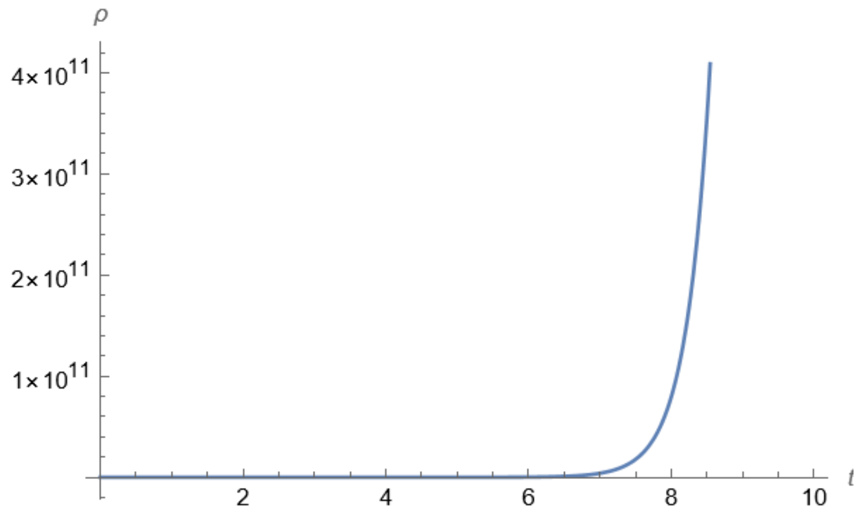

Figure 4 – Change of energy density ρ over time t

As can be seen in Figure 3, the pressure p has a negative value, which indicates the expansion of the Universe. Figure 4 shows that the energy density has a positive value.

In the decelerated phase, when cold dark matter or dusty liquid dominates, the state parameter is defined as $\omega = 0$, and in the radiation epoch $0 < \omega < 1/3$ and in the rigid liquid ω = 1. In the accelerated phase at the cosmological constant or in the vacuum era $\omega = -1$, in the quintessence and quintom epoch $-1 < \omega < -1/3$

.

Figure 5 shows the graphical behavior of the state parameter of the investigated model of modified Gauss-Bonnet gravity. This figure clearly shows that the parameter of the equation of state changes tending to a negative value in the range $-1 \leq \omega \leq 0$, which shows good agreement with observational data of type Ia supernovae.

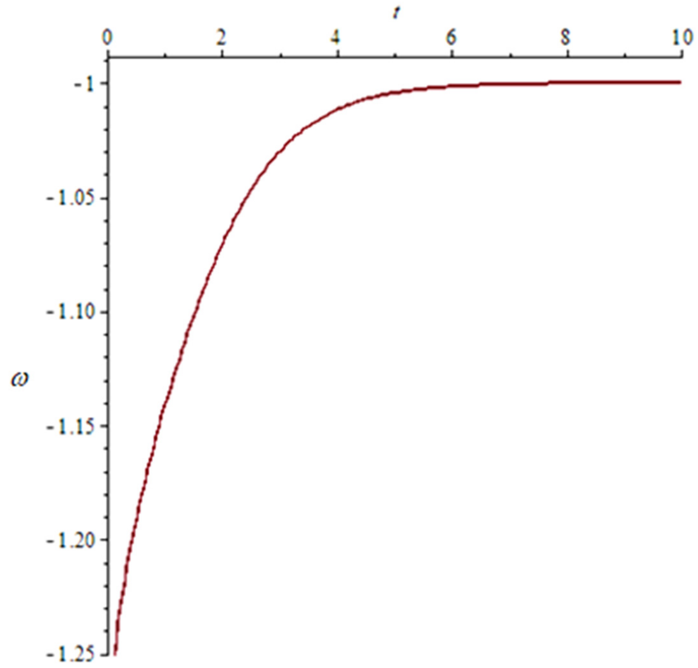

Figure 5 – Change of state parameter ω over time t

4. Conclusion

In this paper the problem of cosmic acceleration in modified $f(R)$, $\eta(G)$ Gauss-Bonnet gravity is investigated. The dependences of the physical quantities $f(R)$, $\eta(G)$, the scale factor, and the Hubble parameter on the cosmological time t are obtained. We found the function $f(R)$ dependent on the curvature scalar R. We also carried out a physical analysis of the obtained solution of the scale factor and found that this model has an adequate cosmological interpretation similar to the de Sitter cosmological model. The found values of the Hubble parameter and scale factor describe the exponential expansion of the Universe, which is shown in Figure 2. Figure 5 shows that the state parameter starts near zero at the beginning of cosmic time, i.e., the Universe is dominated by matter. Then at the end of cosmic time it progresses to a near negative value of -1, which exhibits a vacuum era-like behavior. As a result, our research model is realistic. Thus, it is shown that this modified Gauss-Bonnet gravity model describes the acceleration of the Universe.

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