

Correlation analysis of the wind acceleration parameter in high-mass X-ray binaries

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This work presents a correlation analysis of the wind acceleration parameter β in high-mass X-ray binaries (HMXBs). The aim of the study is to determine how β depends on the geometric and dynamical characteristics of the system, primarily on the ratio of the orbital radius to the stellar radius, a/R_* , and on the terminal wind velocity v_∞ . For this purpose, published values of R_* , a and v_∞ together with measured wind velocities in the vicinity of the neutron star, were used. Based on these data, the parameters of the β -velocity law were determined numerically. The resulting β values were then approximated using a linear regression, $\beta \approx k a/R_* + c$, yielding a simple empirical relation. The results show that the linear correlation between β and the orbital-to-stellar radius ratio is weak (with a Pearson correlation coefficient $r \approx 0.22$) and is accompanied by substantial scatter, indicating the influence of additional physical factors. The linear regression yields a small positive slope ($k \approx 0.14$), which suggests only a weak tendency for β to increase with the orbital ratio. Because of the small sample size, this trend should be regarded as tentative. The proposed linear model allows β to be estimated without detailed spectroscopic analysis and may be useful as a rough empirical indicator of wind-accretion properties in HMXBs. The originality of this study lies in providing a simple empirical first-order assessment of the β parameter in HMXBs based on observed orbital and wind properties in systems where detailed spectroscopic constraints are limited.

Keywords: high-mass X-ray binaries (HMXBs), β -law (β -velocity profile), wind velocity profile, terminal wind velocity, orbital ratio a/R_* , correlation analysis.

1. Introduction

High-mass X-ray binaries (HMXBs) are close binary systems in which a neutron star accretes matter from the radiatively driven stellar wind of a massive early-type companion (O–B/Oe–Be). Since the density and velocity of the stellar wind at the neutron-star orbit determine the efficiency of mass capture, these parameters directly affect the accretion regime, the X-ray luminosity, and the spin evolution of the compact object [1, 2]. Despite the large number of observational studies of HMXBs, the detailed shape of the wind acceleration profile (i.e. the parameters of the β -law) remains poorly constrained. This uncertainty complicates reliable modelling of wind accretion and hampers predictions of neutron-star spin evolution.

The wind velocity as a function of distance from the surface of the donor star is commonly described by the so-called β -law (or CAK-type law):

$$v(r) = v_\infty \left(1 - b \frac{R_*}{r}\right)^\beta,$$

where v_∞ is the terminal wind velocity, R_* is the radius of the donor star, b is a parameter that determines the initial wind speed at the stellar surface, and β characterizes the smoothness of the acceleration. Physically, the parameter β controls how rapidly the outflow reaches v_∞ larger β values correspond to slower, more gradual acceleration, leading to higher wind densities near the neutron star and, consequently, to subsonic (settling) accretion and neutron-star spin-up. In contrast, smaller β values (steeper acceleration) reduce the gas density at the orbital radius and may trigger a transition to the propeller regime, accompanied by neutron-star spin-down. Thus, even relatively small variations in β can significantly alter the accretion regime and the long-term spin evolution of the neutron star [3, 4].

The terminal wind velocity v_∞ is usually determined from ultraviolet spectral lines; however, the parameters b and β are much more difficult to constrain observationally, especially under conditions of strong X-ray irradiation and wind clumping. Moreover, recent studies indicate that the winds of donor stars in HMXBs differ substantially from those of isolated OB stars due to photoionization effects, wind inhomogeneities, and interactions with the orbiting neutron star. Consequently, standard wind parameters derived for single stars cannot be applied directly to HMXBs, and empirical estimates of wind acceleration in individual systems are required to properly understand the accretion processes [5, 6]. This need for empirical constraints is reinforced by recent system-by-system reconstructions of β -law wind profiles in HMXBs, where the parameters b and β were inferred directly from observed terminal and local wind velocities, showing that slowly accelerating, denser winds tend to correspond to larger β values under strong X-ray photoionization [7].

The aim of the present work is to investigate the correlation between the wind acceleration parameter β and the geometric and dynamical properties of high-mass X-ray binaries. In particular, we examine how β depends on the orbital-to-stellar radius ratio a/R_* and on the terminal wind velocity v_∞ . To this end, we use published measurements of R_* , a and v_∞ together with estimates of the local wind velocity at the neutron-star orbit, and numerically determine the parameters b and β . We then construct a linear regression of β as a function of a/R_* and evaluate the correlation between β and v_∞ . This approach yields a simple empirical estimate of β that does not require detailed spectral analysis, thereby providing a practical tool for the rough classification of accretion regimes and for qualitative predictions of neutron-star spin evolution.

The relevance of this study is supported by recent review works. Recent studies have emphasized the combined roles of radiative driving, photoionization, hydrodynamics, wind clumping, and neutron-star feedback in HMXB winds [2, 6, 8]. The systematic empirical determination of the parameter β presented in this work, based on a correlation analysis, helps to bridge the gap between observations and theory and provides new constraints for future studies.

2. Materials and methods

2.1 Determination of the parameters b and β .

For each HMXB considered in this study, published values of the donor-star radius R_* , the orbital semi-major axis a , the terminal wind velocity v_∞ and the measured wind velocity $v(a)$ in the vicinity of the neutron star are used. The SED-fitting method for determining the parameters of hot supergiants can serve as an independent source of donor-star parameters (T_{eff} , $\log g$, A_V), which may be used as input data for constructing relationships between wind properties [9]. The parameters b and β of the wind velocity law given by Eq. (1).

$$v(r) = v_\infty \left(1 - b \frac{R_*}{r}\right)^\beta \quad (1)$$

are determined such that, for given v_∞ , R_* and a the model wind velocity $v(a)$ reproduces the observed value.

The approach observational data $\rightarrow$ model fitting $\rightarrow$ parameter extraction in X-ray binary systems is methodologically consistent with the reconstruction of the β -parameter based on published HMXB data [10,11]. The calculation procedure is as follows:

- The stellar radius R_* and the orbital separation a are converted from solar radii to astronomical units (AU). We adopt the relation $1 R_\odot = 0.00465 \text{ AU}$; thus, $a/R_* = (a/\text{AU}) / (R_*/R_\odot) \cdot 0.00465^{-1}$.
- An initial estimate of the photospheric wind velocity v_0 is adopted, typically $v_0 = 10\text{--}20 \text{ km/s}$.
- From the boundary condition $v(R_*) = v_0$, applied to Eq. (1) the parameter b is expressed as:

$$b = 1 - \left(\frac{v_0}{v_\infty}\right)^{1/\beta} \quad (2)$$

- For a trial value of β , the corresponding b is computed using the above expression and substituted into $v(a) = v_\infty \left(1 - b \frac{R_*}{a}\right)^\beta$. The value of β is then determined numerically by requiring the model velocity $v(a)$ to match the observed value. This is achieved using a root-finding procedure (the root scalar function from the *SciPy* library). Once β is

obtained, the parameter b is recalculated accordingly. This procedure is repeated for all systems in the sample. In addition, the input parameters are varied within their observational uncertainties in order to assess the sensitivity of the derived values of b and β .

2.2 Correlation Analysis.

Pearson correlation analysis is a standard statistical tool widely used in astrophysical studies. In this work, it is employed to investigate the dependence of the wind acceleration parameter β on the system geometry and wind velocity in HMXBs [12]. To search for a statistical relationship between the wind acceleration parameter β and the orbital ratio a/R_* , the Pearson correlation coefficient is calculated using Eq. (3):

$$r_{xy} = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_i (x_i - \bar{x})^2 \sum_i (y_i - \bar{y})^2}}, \quad (3)$$

where $x_i = a/R_{*,i}$, $y_i = \beta_i$, $\bar{x}$ and $\bar{y}$ — denote the mean values of x and y over the sample. To approximate the dependence of β on the orbital-to-stellar radius ratio, a linear regression of the form

$$b \approx k \frac{a}{R_*} + c$$

is constructed, where the coefficients are determined as:

$$k = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}, \quad c = \bar{y} - k\bar{x}.$$

The obtained coefficients k and c characterize the slope and intercept of the regression line. In the present study, the sign and magnitude of k are used only to describe the direction and strength of the empirical trend.

2.3 Error Estimation.

Uncertainties in the input parameters (R_* , a , v_∞ , $v(a)$) propagate into the derived results. To assess the sensitivity of the method, the input parameters are repeatedly varied at random within their observational uncertainties. For each realization of the input data, the corresponding values of b and β are computed. The resulting distributions are then used to estimate the mean values and variances of b and β . This approach makes it possible to evaluate

the robustness of the results against measurement errors and to identify which parameters most strongly contribute to the uncertainty in β .

2.4 The systems considered.

CenX-3

Cen X-3 was discovered in 1967 by the *Uhuru* space observatory as a point-like X-ray source. Subsequent observations revealed regular X-ray pulsations with a period of $P_s = 4.876 \pm 0.0154$ s, which allowed the source to be identified as an X-ray pulsar [1]. The detection of sinusoidal intensity variations with a period of ~ 2.087 days further demonstrated that Cen X-3 is an eclipsing binary system with a massive companion. Optical observations of the Cen X-3 field performed with telescopes at the ESO and Las Campanas observatories during 1973–1974 led to the identification of the optical counterpart as an early-type star with a spectral classification of O9–O9.5 V or B0 I–III. As a result, Cen X-3 became the first object firmly identified as an X-ray pulsar in a massive binary system. Subsequent optical studies refined the spectral type of the donor [13] star to O6.5 II–III, with a mass of approximately $\sim 20.2 M_\odot$ located at a distance of $d_0 \approx 5.7 \pm 1.5$ kpc [14]. The orbital eccentricity of the compact object is extremely small, $e \approx 0.00021$, indicating an orbit that is nearly circular. The terminal wind velocity is $v_\infty \sim 2000$ km/s [15].

OAO1657-415

OAO 1657–415 was initially discovered in 1978 as an eclipsing X-ray source and was identified one year later as an accreting X-ray pulsar with a spin period of 38.22 s at that time. The X-ray luminosity of the source in the 18–50 keV energy band during the 2002–2016 observations varied smoothly within the range $L_x \approx 0.1 - 2 \times 10^{37} (d/d_0)^2$ erg/s [16]. OAO 1657–415 is located at a distance of $d_0 = 6.4 \pm 1.5$ kpc and is identified as a high-mass X-ray binary system with an orbital period of $P_{orb} \approx 10.44$ days [17]. The orbital eccentricity is $e \approx 0.104$. The system consists of a neutron star and a massive supergiant companion of spectral type Ofpe [18]. The neutron star currently rotates with a spin period of $P_s \approx 37.024$ s and possesses a surface magnetic field strength of $B_{NS} = (3.1 \pm 0.2) \times 10^{12}$ G, as inferred from the analysis of a cyclotron resonance scattering feature detected in the X-ray spectrum of the source. The terminal wind velocity is $v_\infty \sim 500$ km/s [19].

VelaX-1

Vela X-1, like Cen X-3, was originally discovered by the *Uhuru* space observatory during the early years of X-ray astronomy in 1967 and was subsequently identified as an X-ray pulsar in a massive binary system. The spin period of the compact object is $P_s \sim 283,43$ s [20]. The orbit has a relatively small eccentricity, close to circular, $e \sim 0,088$, which is characteristic of quasi-stationary wind-fed pulsars. The orbital period is $P_{orb} \sim 8,96$ days [17]. The neutron star is accompanied by the supergiant star HD~77581, located at a distance of $d_0 = 1,9 \pm 0,2$ kpc and classified as a B0.5~Ia star with an estimated mass of $M_* \sim 26 M_\odot$ [17]. Based on optical and ultraviolet spectroscopic studies, terminal wind velocity is $v_\infty = 1100$ km/s and a mass-loss rate of $\dot{M} \sim 4 \times 10^{-6} M_\odot/\text{yr.}$ have been reported [21].

4U1538-52

4U~1538-52 was discovered in 1976 by the *OSO-8* space observatory as an eclipsing X-ray source in a binary system. Subsequent observations carried out during the *Ariel-5* mission revealed X-ray pulsations with a period of $\sim 528,93$ s, associated with the axial rotation of the neutron star. At the present epoch, the spin period of 4U~1538-52 is $P_s \simeq 526,23$ s [22], while the orbital period is $P_{orb} \simeq 3,728$ days [17]. Optical spectroscopic observations performed with the 3,9-m telescope at the Siding Spring Observatory (Australia) allowed the identification of the normal companion (QV~Nor) as an early-type supergiant of spectral class B0.2~Ia, with an estimated mass-loss rate of $\sim 10^{-6} M_\odot/\text{yr.}$ [23]. The distance to the source is estimated to be $d \simeq 5 \pm 0,5$ kpc. The terminal wind velocity is $v_\infty \sim 1450$ km/s [24].

GX301-2

GX~301-2 is a unique object among the population of quasi-persistent X-ray pulsars in high-mass binary systems, as it possesses an orbit with a relatively large eccentricity $e \simeq 0,46$ [25], which is more typical of transient X-ray pulsars. In the vast majority of quasi-persistent systems, the orbital eccentricity does not exceed 0,2. The distance to the source is estimated to be $3,5 \pm 0,5$ kpc. The normal component of the system is the supergiant star BP~Cru [5], classified as an early-type B1.5~Ia star with an estimated mass of $M_* \sim 48 M_\odot$ [16], making it one of the most massive donor stars among the

Galactic population of X-ray pulsars in high-mass X-ray binaries. Optical spectroscopic studies of the donor star performed with the ESO 3,6-m telescope (Chile) revealed prominent P~Cygni profiles, which allowed estimates of the mass-loss rate in the range $\dot{M} \sim 5 \times 10^{-6} - 10^{-5} M_\odot/\text{yr.}$ with a terminal wind velocity of $v_\infty \sim 400$ km/s [5].

2S0114+650

2S~0114+650 was discovered in 1977 during an all-sky survey carried out by the *SAS-3* X-ray observatory. Several months after its identification, it was established that the compact source is a member of a binary system with a massive companion, the star LS~I~+65~010 (V662~Cas). Based on its observational properties, 2S~0114+650 is classified as a quasi-persistent X-ray pulsar with an exceptionally long spin period of $P_s \sim 9475$ s, making it the second slowest-spinning pulsar among the Galactic population of HMXBs. Since its discovery, 2S~0114+650 has exhibited a long-term spin-up trend at a rate of $\sim 10^{-6}$ s/s, although the physical origin of its extremely long spin period remains unclear. The current orbital period of the system is $P_{orb} \simeq 11,591$ days. The massive companion of the neutron star is the supergiant V662~Cas, classified as a B1~Ia star [17]. Detailed optical spectroscopic studies of V662~Cas conducted with the 2,5-m Isaac Newton Telescope at the La Palma Observatory yielded estimates of the terminal wind velocity $v_\infty \simeq 1200 \pm 300$ km/s and a mass-loss rate of $\dot{M} \sim 3 \times 10^{-6} M_\odot/\text{yr.}$ [26].

3. Results

3.1 Input data and derived parameters.

Table 1 lists the input parameters (R_* , a , v_∞ , $v(a)$) for each HMXB considered, together with the derived parameters b , β and the orbital ratio a/R_* . The values of the local wind velocity $v(a)$ were taken from published X-ray observations [27], while the terminal wind velocities v_∞ were adopted from ultraviolet spectroscopy. The stellar radii R_* and orbital separations a were obtained from optical and dynamical measurements. The parameter b was calculated using $b = 1 - (v_0/v_\infty)^{1/\beta}$ assuming a photospheric wind velocity $v_0 = 20$ km/s, whereas the parameter β was determined numerically by solving the equation $v(a) = v_\infty \left(1 - b \frac{R_*}{a}\right)^\beta$.

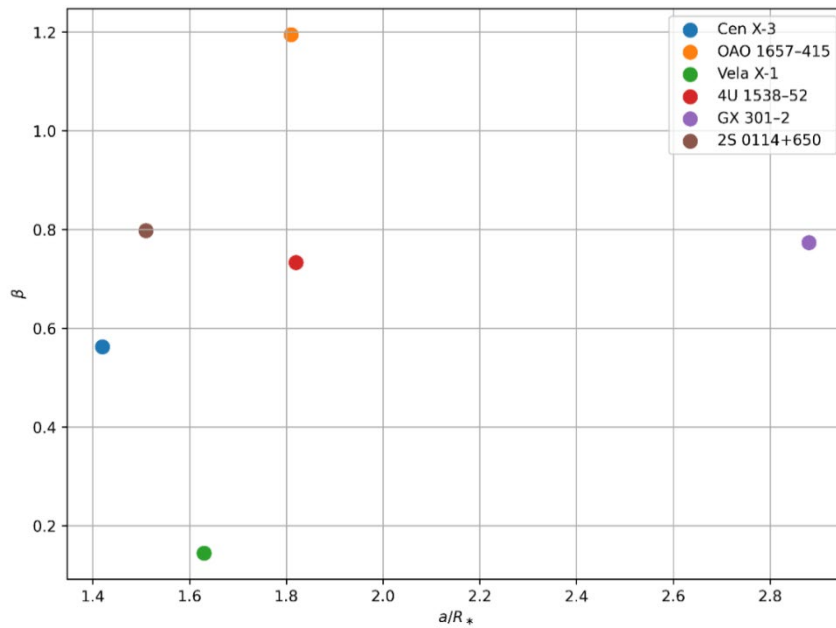
Table 1. Input data and derived parameters for the HMXB system.

Systems	$R_*(R_\odot)$	$a(AU)$	$v_\infty(km/s)$	$v(a)(km/s)$	a/R_*	b	β
Cen X-3	12.1	0.08	2000	1010	1.42	0.999	0.5625
OA0 1657-415	25.0	0.21	500	210	1.81	0.933	1.1948
Vela X-1	30.4	0.23	700	610	1.63	1.000	0.1443
4U 1538-52	13.0	0.11	1450	810	1.82	0.997	0.7333
GX 301-2	62.0	0.83	400	290	2.88	0.979	0.7735
2S 0114+650	37.0	0.26	1200	510	1.51	0.994	0.7978

3.2 Plot of β as a function of a/R_* .

Figure 1 presents the β versus a/R_* diagram for the systems considered in this study. The points correspond to the values listed in Table 1 and indicate only a weak correlation between the orbital-to-stellar radius ratio and the wind acceleration parameter β . The Pearson correlation coefficient

between β and a/R_* is $r \approx 0.22$, indicating a weak positive correlation accompanied by a substantial scatter in the data. Uncertainty bars are not shown in Figures because the available input uncertainties are heterogeneous across the selected systems; the plot is therefore intended only for qualitative visualization of the trend.

**Figure 1.** Dependence of the parameter β on a/R_* for HMXBs.

3.3 Plot of β as a function of terminal wind velocity v_∞ .

Figure 2 shows the dependence of the wind acceleration parameter β on the terminal wind velocity v_∞ . Overall, the diagram does not reveal a clear linear relationship between β and v_∞ and the data exhibit substantial scatter. Nevertheless, systems with relatively low v_∞ may show higher β ,

corresponding to a more gradual wind acceleration, whereas faster winds tend to be associated with smaller β values. This behavior is physically plausible, although the small sample size and the presence of outliers indicate that additional physical factors (e.g. X-ray photoionization, clumping, and binary interaction effects) likely play an important role.

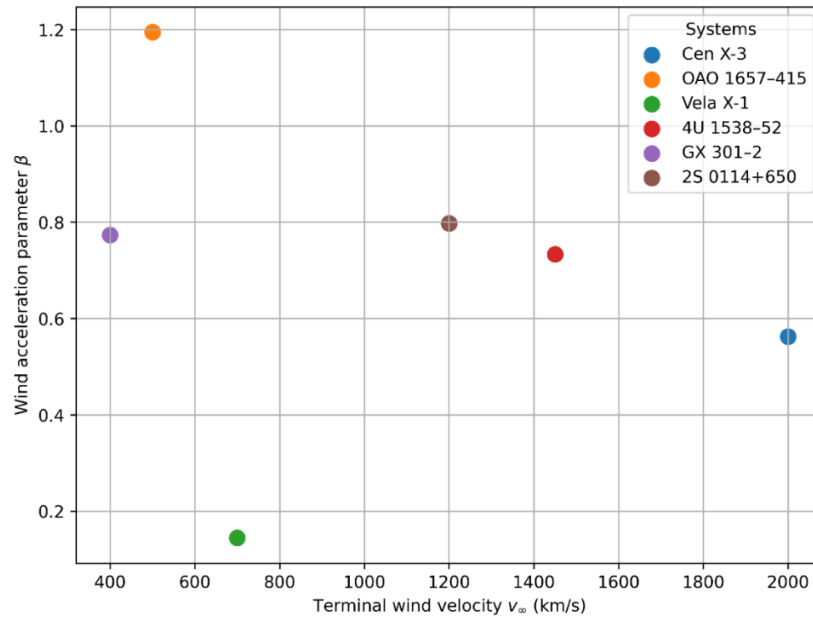


Figure 2. Dependence of the parameter β on the terminal wind velocity v_∞ . The color of each point corresponds to a specific system; see the legend inside the figure.

3.4 Discussion of correlation parameters

A numerical approximation in the form of a linear relation $\beta(a/R_*)$ yields coefficients $k \approx 0.14$ and $c \approx 0.44$. The positive but small value of k indicates a very weak tendency for β to increase with the orbital-to-stellar radius ratio a/R_* . The Pearson correlation coefficient $r \approx 0.22$ confirms that the linear dependence between β and this geometric orbital parameter is weak and not statistically significant. This suggests that, in addition to the system geometry, the wind acceleration parameter β is strongly influenced by other physical factors, such as the donor star's spectral type, the level of X-ray photoionization, the neutron star's magnetic field, and wind inhomogeneity. Nevertheless, the linear model may be considered only as a preliminary empirical approximation. Higher β values are generally associated with slower wind acceleration and higher wind density near the neutron star, whereas lower β values correspond to faster acceleration. However, given the weak correlation and the limited sample size, the present analysis does not allow firm conclusions about specific accretion regimes or spin evolution pathways.

4. Discussion

4.1 Physical interpretation of the β correlation

Given the small sample size and the exploratory nature of the present study, a linear regression was

adopted only as a first-order empirical approximation. More complex nonlinear dependencies may exist, but their assessment requires a larger sample and is beyond the scope of this work. The analysis of the dependence of the wind acceleration parameter β on the orbital ratio suggests at most a weak positive tendency for β to increase with increasing orbital-to-stellar radius ratio. However, this trend is accompanied by substantial scatter and is not statistically significant. Therefore, the observed relation should be interpreted with caution. A possible interpretation is that the wind acceleration parameter is influenced not only by orbital geometry, but also by several additional factors, including X-ray photoionization, wind clumping, donor spectral type, and magnetic interaction effects. For this reason, the orbital ratio alone cannot be considered a reliable predictor of β . The present result should be treated as a preliminary empirical trend that requires confirmation with a larger sample.

4.2 Connection between β and accretion regimes

It is well known that large values of β (of order unity and above) correspond to smooth, slow wind acceleration. Physical effects affecting X-ray emission in the vicinity of a neutron star are discussed; this context is important for interpreting wind accretion and potential regime transitions in HMXBs [28]. In this case, the wind velocity at the neutron-star orbit remains significantly below the terminal

value, leading to a larger effective capture radius and favoring subsonic (settling) accretion. Such accretion is associated with a relatively stable transfer of angular momentum, resulting in neutron-star spin-up. This interpretation is also consistent with recent statistical comparisons of persistent and transient HMXBs, which show that differences in X-ray luminosity behaviour and variability are closely related to the underlying accretion conditions and wind-feeding properties of the systems [29]. The theoretical framework for modeling the rotation of compact objects provides a basis for discussing the spin evolution of an accreting neutron star in HMXBs [30, 31]. In the present sample, the derived β values suggest possible differences in wind acceleration among the analyzed systems. The relatively high value obtained for OAO~1657–415 ($\beta \approx 1.2$) is broadly consistent with slower wind acceleration, whereas the very low value found for Vela~X-1 ($\beta \approx 0.14$) may indicate more rapid acceleration. Intermediate β values (~ 0.6 – 0.8) obtained for Cen~X-3, 4U~1538–52, GX~301–2, and 2S~0114+650 are compatible with moderately accelerating winds. Nevertheless, these associations should be treated as qualitative rather than predictive, because the present sample is small and the inferred correlation remains weak and statistically insignificant.

4.3 Limitations of the study

It should be emphasized that our sample is small (six systems) and does not include Be/X-ray binaries. Therefore, the statistical significance of the inferred correlations is low (with a Pearson correlation coefficient $r \approx 0.22$). In addition, the model is based on the simplified β -law and does not account for the effects of photoionization, magnetohydrodynamic processes, or wind clustering (clumping). These factors can significantly modify the actual wind acceleration, particularly in systems subject to strong X-ray irradiation. It should also be noted that the derived values of β and b are sensitive to uncertainties in the input parameters (R_* , a , v_∞ , $v(a)$); more accurate observations may change the shape of the inferred dependence.

4.4 Comparison with theory and observations.

Our results are generally consistent with theoretical expectations: classical models of radiatively driven winds in single OB stars predict $\beta \sim 0.5$ – 1.0 whereas in HMXBs higher β values may arise due to X-ray irradiation and orbital interaction effects. Recent analytical work on generalized β -velocity

laws in X-ray-illuminated HMXBs has shown that the allowed parameter space of b and β is physically constrained by the requirement of a smooth monotonic CAK-type outflow, with a robust lower bound $\beta \geq 1/2$, while strong X-ray irradiation may favor larger β values [32]. In the present sample, the fitted linear relation yields only a small positive slope, and the correlation remains weak and statistically insignificant. Therefore, the current data do not support a firm monotonic dependence of β on orbital geometry alone. A comparison with previous studies shows that our estimated β values (0.14–1.19) lie within the broad range reported for donor-star winds and wind-fed accretion systems. In particular, systems with relatively low v_∞ and high β , such as OAO 1657–415, are broadly consistent with denser and more slowly accelerating winds, whereas systems with higher v_∞ and lower β may be associated with more rapidly accelerating and less dense outflows. However, these interpretations should be regarded as qualitative, since the present sample is limited and the inferred trend is not statistically strong.

Future work based on an expanded sample, including Be/X-ray binaries, will be necessary to clarify whether any systematic dependence of β on orbital parameters is physically meaningful. Investigating orbital-phase variations of β and incorporating photoionization effects may also help to develop more accurate models capable of reproducing the observed behavior of X-ray luminosity and neutron-star spin evolution.

5. Conclusion

In this work, we proposed and implemented a simple linear model relating the wind acceleration parameter β to the orbital-to-stellar radius ratio a/R_* . The correlation analysis shows that the relationship between a/R_* and β is weak ($r \approx 0.22$) and accompanied by substantial scatter, indicating that the dependence is not statistically significant. The fitted empirical relation, $\beta \approx 0.14 (a/R_*) + 0.44$, therefore represents only a preliminary approximation rather than a robust physical law.

The proposed model may be useful for obtaining rough first-order estimates of β in systems where detailed spectroscopic constraints are not available. However, due to the limited sample size and the weak correlation, the relation should be applied with caution and should not be used for quantitative predictions of accretion regimes or neutron-star spin evolution without additional supporting evidence.

The results suggest that the wind acceleration parameter β is influenced not only by orbital geometry but also by other physical factors, such as X-ray photoionization, wind clumping, and magnetic interactions. Therefore, the orbital-to-stellar radius ratio alone cannot serve as a reliable predictor of wind acceleration properties in high-mass X-ray binaries.

Future work should focus on expanding the sample of sources, including systems with Be-type companions, in order to improve the statistical significance of the analysis. It is also important to incorporate additional physical processes into the modeling and to investigate possible orbital-phase and long-term variations of β and related parameters. Such studies will help to clarify the role of wind structure in accretion processes and to develop more accurate models of mass transfer and spin evolution in high-mass X-ray binaries.

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Conflicts of interest

The authors declare no conflict of interest.

Author contributions

Conceptualization, Z.M. and S.T.; Methodology, Z.M., S.T. and N.A.; Formal Analysis, Z.M. and N.A.; Investigation, Z.M., A.M. and N.A.; Data Curation, Z.M. and A.M.; Visualization, A.T. and Z.M.; Writing – Original Draft Preparation, Z.M.; Writing – Review & Editing, S.T., A.M., A.T. and N.A.; Validation, S.T., A.T. and N.A.; Supervision, S.T. All authors have read and agreed to the published version of the manuscript.

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