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Dynamical collision frequency and conductivity of dense plasmas

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In order to obtain the dynamical collision frequency and the dynamical conductivity, we used the molecular dynamics simulation of dense two-component plasmas with the pair interaction potential of charged particles taking into account quantum-mechanical effects.The temperature range of $10^4 K < T < 10^8 K$ and the density range of $10^{21} cm^{-3} < n \leq 10^{24} cm^{-3}$ were considered. It has been shown that at high temperatures the results for the static collision frequency arein a good agreement with the well-known models of ideal plasmas.It has been found that the dynamical collision frequency of electrons is nearly constant at frequencies lower than the electron plasma frequency and drops fast at frequencies higher than the electron plasma frequency. It is also shown that ina field with a frequency higher than the electron plasma frequency the dense plasma behaves like an insulator in terms of conductivity.

Key words: non-ideal plasmas, molecular dynamics method.
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1 Introduction

The study of the plasma state is stimulated by the fundamental interest in understanding its nature and importance of its practical use. It is well known that the source of energy in stars is thermonuclear reactions. An idea of a controlled thermonuclear fusion has led to the intensive research in the field of plasma physics. At the present time the laboratory plasmas are investigated in a wide range of temperatures and pressures, from a low-temperature gas-discharge plasma up to the dense plasma obtained by the laser beam [1], [2]. Some of the reasons for a great interest in the investigation of the dense plasma properties are discussed in [3-5].

This work is devoted to the study of the dynamical collision frequency of electrons in two-component dense plasmas. For the considered plasma parameters the thermal wavelength of electrons becomes comparable with the mean inter-particle distance. In a molecular dynamics simulationwe used the following quantum potential correctly taking into account the quantum diffraction effect in dense plasmas [6]:

$$u_{ab}(r) = \frac{z_a z_b e^2}{r}\left\{1 - \tanh\left(\sqrt{2}\frac{\lambda_{ab}^2}{a^2 + br^2}\right)e^{-\tanh(\sqrt{2}\lambda_{ab}^2/(a^2+br^2))}\right\}\left(1 - e^{-r/\lambda_{ab}}\right) \quad (1)$$

where $\lambda_{ab} = \hbar / \sqrt{4\pi m_{ab} k_B T}$ is the thermal wavelength, $m_{ab} = m_a m_b / (m_a = m_b)$, $a = (3/4\pi n)^{1/3}$ is the average inter-particle distance and $b = 0.033$.

In the limit $T \to \infty$ the potential (1) coincides with the Deutsch potential [7]:

$$u_{ab}\big|_{T\to\infty} = \frac{z_a z_b}{r}\left(1 - \exp[-r/\lambda_{\alpha\beta}]\right) \quad (2)$$

Figures 1-2 show the quantum potentials (1) and (2), where $\Gamma = \beta e^2 / a$ (here the density parameter r_S, which is the ratio of the mean inter-particle distance to the first Bohr radius,is introduced). As it is seen the quantum potential (1) gives weaker interaction than the potential (2).

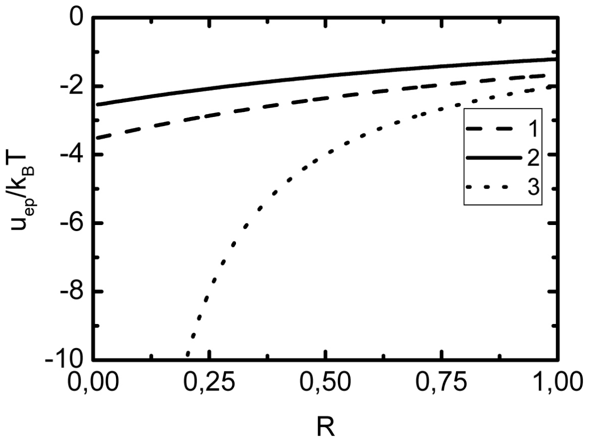

Figure 1 – Proton-electron pair interaction potential. Curve 1 is the Deutsch potential (11), curve 2 is thesemiclassical potential (10), curve 3 is the Coulomb potential, where $R = r / a_0$, $\Gamma = 2$, $r_S = 1$

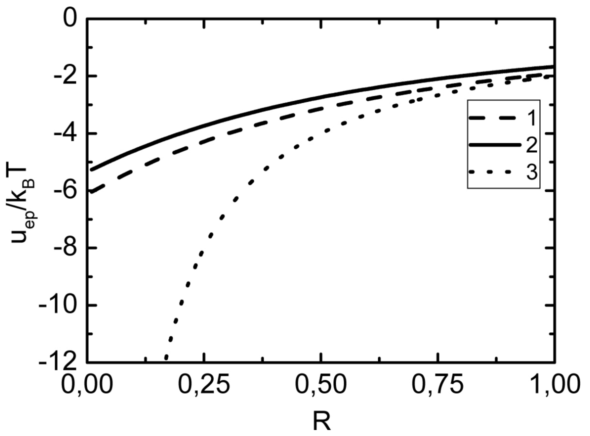

Figure 2 – Proton-electron pair interaction potential. Curve 1 is the Deutsch potential (11), curve 2 is the semiclassical potential (10), curve 3 is the Coulomb potential, where $R = r / a_0$, $\Gamma = 2$, $r_S = 3$

The potential (1) can be used for plasma at low temperature $T < 10^4 K$ if its density is sufficiently high and plasma is fully ionized due to the Mott transition.

2 The dynamical collision frequency and the dynamical conductivity

As it is well known, the characteristic collective oscillation frequency in the plasma is defined by the plasma frequency:

$$\omega_P = \sqrt{\frac{4\pi n_e e^2}{m_e}} . \quad (3)$$

In the plasma, an incident beam experiences strong dispersion near the plasma frequency ω_P.

A full induced electric field $\vec{D}$ is related to the field intensity $\vec{E}$ according to the following formula:

$$D_i(\omega, k) = \varepsilon_{ij}(\omega, k) E_j(\omega, k), \quad (4)$$

where the dielectric tensor (DT) depends on the frequency ω and the wave vector k:

$$\varepsilon_{ij}(\omega, k) = \int_0^\infty d\tau \int d\vec{r} \exp\left(-i\left(\omega t - \vec{k}\vec{r}\right)\right)\varepsilon_{ij}(\mathrm{t}, \mathrm{r}).$$

(5)

The dependence of the $\varepsilon_{ij}(\omega, k)$ on the wave vector corresponds to the correlation of the field values in space. In plasmas, the radius of the volume, around the given point, within which the field value at thegiven point strongly correlates with that in the other points is defined by the screening length. As the screening length in the classical plasma,one should takeeither the Debye radius $r_D = \sqrt{4\pi ne^2 / k_B T}$ (Tomas-Fermi screening length in degenerate plasmas) or the mean free path, depending on which of them is smaller. In dense plasmas the mean inter-particle distance $a = (3/4\pi n)^{1/3}$ can be taken as the characteristic screening length.

The DT can be expressed in terms of the conductivity tensor:

$$\varepsilon^{l,tr}(\omega, k) = 1 + i\frac{4\pi}{\omega}\sigma^{l,tr}(\omega, k). \quad (6)$$

$\varepsilon_{ij}(\omega, k)$ can be also divided into longitudinal and transversal parts:

$$\varepsilon_{ij}(\omega, k) = \frac{k_i k_j}{k^2}\varepsilon^l(\omega, k) + \left(1 - \frac{k_i k_j}{k^2}\right)\varepsilon^{tr}(\omega, k)$$

The longitudinal part is related to the density oscillations. The imaginary part of the $\varepsilon^l(\omega,k)$ describes the energy dissipation and the real part of the $\varepsilon^l(\omega,k)$ describes density fluctuations. Moreover, within the linear response theory the longitudinal part of the DT is the response function and determines all electrodynamic properties of the plasma. Therefore, further the longitudinal part of the DT is considered. For the purposes of convenience the longitudinal part of the DT is denoted as simply $\varepsilon(\omega,k)$ and referred to as the dielectric permeability (DP).

In practice, an incident radiation has the wavelength much greater than the screening length. Therefore, we did not consider the spatial dispersion of the $\varepsilon(\omega,k)$ [8], we only studied its dependence on frequency.

The DP and the dynamic conductivity can be obtained using the well-known Drude-Lorentz formulas:

$$\varepsilon(\omega)=1-\frac{\omega_P^2}{\omega[\omega+i\nu(\omega)]}, \tag{8}$$

$$\sigma(\omega)=\frac{\omega_P^2/4\pi}{\nu(\omega)-i\omega}, \tag{9}$$

where the dynamical collision frequency $\nu(\omega)$ was introduced.

From the Drude-Lorentz formula it is seen that electrodynamic properties can be investigated either directly by obtaining a dynamic DP or alternatively through the dynamical collision frequency $\nu(\omega)$.

According to the Green-Kubo theory the dynamic DP can be obtained if the generalized susceptibility $\alpha(\omega)$ is known [9,10]:

$$\varepsilon^{-1}(\omega)=1-i\frac{4\pi}{\omega}\alpha(\omega). \tag{10}$$

where the susceptibility expressed in terms of the velocity autocorrelation function $\langle\vartheta(0)\cdot\vartheta(t)\rangle$ of electrons:

$$\alpha(\omega)=\frac{e^2n_e}{3k_Б T}\int_0^\infty\langle\vartheta(0)\cdot\vartheta(t)\rangle e^{i\omega t}dt. \tag{11}$$

From equations (8) and (10) the dynamical collisions frequency can be expressed in terms of the generalized susceptibility:

$$\frac{\nu(\omega)}{\omega_P}=\frac{\omega_P}{4\pi\alpha(\omega)}+i\left(\frac{\omega}{\omega_P}-\frac{\omega_P}{\omega}\right). \tag{12}$$

3 Results

Figure 3 shows the dynamic collision frequency obtained from the molecular dynamics simulation for the given density parameter and different coupling parameters.The star marks the static collision frequency obtained using the analytical formula for the weakly coupled plasma ($\Gamma<1$)[8]. This analytical formula was derived using the scattering cross section for the particles interacting via the Debye potential:

$$\nu=\pi\frac{e^4}{k_BT}\bar{\upsilon}n\ln\left(0.37\frac{k_BT}{e^2n^{1/3}}\right), \tag{13}$$

where $\bar{\upsilon}$ is the most probable velocity of particles.

As it is seen from Fig.3 the dynamical collision frequency tends to the static limit and roughly remains the same at frequencies smaller than the plasma frequency. On the contrary, at frequencies higher than the plasma frequency the collision frequency drops sharply. This is due to the fact that the electron cannot experience many collisions during the time shorter that the time required for passing the mean free path, which is characterized by the inverse value of the plasma frequency. It is also seen that the collision frequency decreases with increasing temperature. This behavior is in agreement with the well-known model of a collisionless plasma, which is applicable at high temperatures $\Gamma<<1$.

Figure 4 shows the dynamical collision frequency for different densities at a given temperature. An increase in density causes an increase in the dynamical collision frequency, which is caused bythe decrease in the mean free path of particles.

In Fig.5 the dynamical conductivity obtained from a molecular dynamics simulation is presented in comparison with its static value and the value obtained from the Spitzer theory. Figure 6 shows the dynamical conductivity for different densities at a constant temperature. An increase in density causes an increase in the dynamical conductivity, which is obviously due to the increase in the number of charged particles.

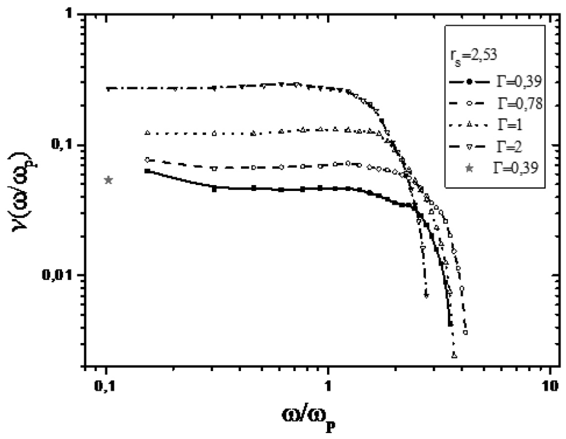

Figure 3 – The dynamical collision frequency at different temperatures

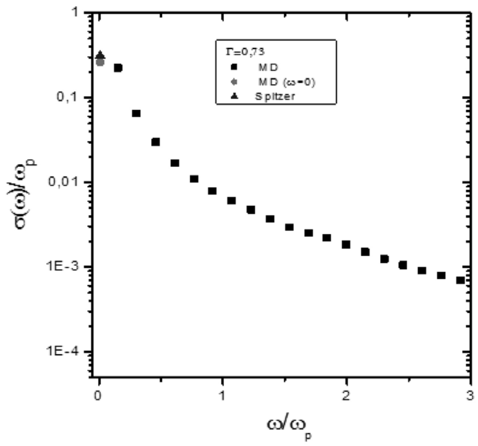

Figure 5 – The dynamical conductivity

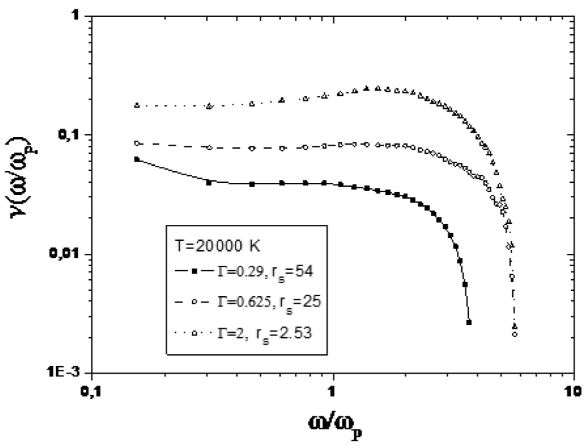

Figure 4 – The dynamical collision frequency at different densities

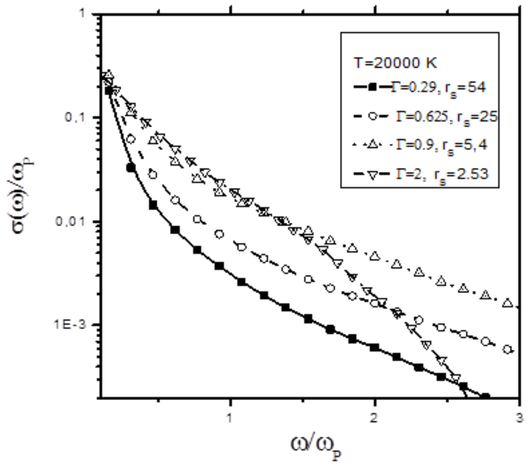

Figure 6 – The dynamical conductivityat different densities

4 Conclusion

The results obtained in this work indicate that the collision frequency does not change strongly in the range of frequencies lower than the plasma frequency and rapidly goes down with increasing frequency for $\omega > \omega_P$. It was also shown that in a high frequency field $\omega >> \omega_P$ the dense plasmas behave like insulators in terms ofelectrical conductivity. This fact is understandable if one bears in mind that conductivity in the plasmais the result of drift of electrons and ions and the alternating field with a high frequency may displace particles at large distancesdue to their inertness.

Acknowledgements

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