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Formation and decay of resonance state in ^9Be and ^9B nuclei. Microscopic three-cluster model investigations

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We study nature of the low-lying resonance states in mirror nuclei ^9Be and ^9B. Investigations are performed within a three-cluster model. The model make uses of the hyperspherical harmonics, which provide convenient description of three-cluster continuum. Much attention is paid to the controversial $1/2^+$ resonance states in both nuclei. We study effects of Coulomb interaction on energy and width of three-cluster resonances in the mirror nuclei ^9Be and ^9B. We look for the Hoyle-analogue states which allows for alternative way of ^9Be and ^9B synthesis in a triple collision of clusters.

Key words: Microscopic model, triple collision, resonance state.
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1 Introduction

Resonance state is one of the challenging problems for theoretical and nuclear physics. There are common features of resonance states, observed in a few- or many-channel systems.However, there are some specific features connected with the way of excitation or generation of resonance statesand also in different way of resonance state decay in nuclear systems. Special attention is attracted by resonance states formed by three interacting clusters, i.e. resonance states embedded in three-cluster continuum. Such resonance states are repeatedly observed in nuclei with well-determined three-cluster structure. These nuclei have dominant three-cluster configuration, it means that bound states and many resonance states are lying bellow and above, respectively, threshold of three-cluster continuum. In other words, bound states and large part of resonance states in three-cluster nuclei are generated by an interaction of three clusters. Asexamples of such nuclei, we can mention ^5H, ^6He and ^6Be, ^9Be and ^9B and many others.

In present paper, a microscopic three-cluster model is applied to study nature of resonance states in ^9Be and ^9B. Dominant three-cluster configurations $\alpha + \alpha + n$ and $\alpha + \alpha + p$, respectively, are selected to describe the low excitation energy region in these nuclei. Microscopic model, which was formulated in [1], make uses of total basis of oscillator functions to describe intercluster motion. The model is called as AM HHB which stands for the Algebraic three-cluster Model with the Hyperspherical Harmonics Basis.The first application of this model to study resonance structure of ^9Be and ^9B was made in Ref. [2]. Results presented in [2] were obtained with the Minnesota potential.In present paper we make use of the modified Hasegawa-Nagata potential, and we pay much more attention to the $1/2^+$ resonance states, the Coulomb effects on resonance states in mirror nuclei. Besides, we look for the Hoyle analogue states in ^9Be and ^9B.

2 Model formulation

In this section we shortly outline main ideas of the model. We start with a wave function of nucleus consisting of three clusters, as this a key element of model formulation. To describe three-cluster system one has to construct a three-cluster function

$$\Psi_{JM_J} = \left\{ \left[\left[\Phi_1(A_1,s_1)\Phi_2(A_2,s_2) \right] \Phi_3(A_3,s_3) \right]_S f_L^{(J)}(\mathbf{x},\mathbf{y}) \right\}_{JM_J} \quad (1)$$

and by solving many body Schrödinger equation one has to determine intercluster wave function $f_L^{(J)}(\mathbf{x},\mathbf{y})$ and spectrum of bound state(s) or *S*-matrix for states of continuous spectrum. Jacobi vectors **x** and **y** determine relative position of clusters. Wave functions $\Phi_\alpha(A_\alpha,s_\alpha)$ (α=1, 2, 3), describing internal motion of cluster consisted of A_α nucleons and with the spin s_α, are assumed to be fixed, they posses some very important features, such as, for instance, they are antisymmetric andtranslation-invariant ones. Adiabaticity, connected with a fixed form of the wave functions $\Phi_\alpha(A_\alpha,s_\alpha)$, is the main assumption of the method which is well-known as the resonating group method [3]. Wave function is projector operator which reduces many-particle problem to three-body problem with nonlocal and energy-dependent potential (see detail in Ref. [3]). For amplitudes

$$f_L^{(J)}(\mathbf{x},\mathbf{y}) = f_{\lambda,l;L}^{(J)}(x_3,y_3)\left\{Y_\lambda(\hat{x})Y_l(\hat{y})\right\}_{LM_L} \quad (2)$$

one can deduce an infinite set of the two-dimension integro-differential equations. This set of equations can be more simplified, if we introduce hyperspherical coordinates $\Omega=\{\theta,\hat{x},\hat{y}\}$

$$x=\rho\cos\theta, \quad y=\rho\sin\theta, \quad (3)$$

and construct full set of orthonormalized hyperspherical harmonics (see definition of the harmonics, for instance, in [4], [1])

$$\mathcal{Y}_{K,l_1,l_2,LM}(\Omega)=\chi_{K,l_1,l_2}(\theta)\left\{Y_\lambda(\hat{x})Y_l(\hat{y})\right\}_{LM_L} \quad (4)$$

then wave function (1) represented as

$$\Psi_{JM_J} = \sum_{K,l_1,l_2,L}\left\{\left[\left[\Phi_1(A_1,s_1)\Phi_2(A_2,s_2)\right]\Phi_3(A_3,s_3)\right]_S \times \psi_{K,l_1,l_2;L}(\rho)\mathcal{Y}_{K,l_1,l_2;L}(\Omega)\right\}_{JM_J}, \quad (5)$$

where hyperradial components $\psi_{K,l_1,l_2;L}(\rho)$ of wave function obey an infinite set of integro-differential equations. Last step toward the simplification of numerical solution of such system of equations is to expand the hyperradial amplitudes $\left\{\psi_{K,l_1,l_2;L}(\rho)\right\}$ over basis of hyperradial part of oscillator functions in six-dimension space

$$\psi_{K,l_1,l_2;L}(\rho)=\sum_{n_\rho}C_{n_\rho K,l_1,l_2;L}(b)R_{n_\rho,K}(\rho,b), \quad (6)$$

where $R_{n_\rho,K}(\rho,b)$ is an oscillator function

$$R_{n_\rho,K}(\rho,b)=(-1)^{n_\rho}\mathcal{N}_{n_\rho,K}r^K\exp\left\{-\frac{1}{2}r^2\right\}L_{n_\rho}^{K+3}(r^2), \quad (7)$$

$$r=\rho/b, \quad \mathcal{N}_{n_\rho,K}=b^{-3}\sqrt{\frac{2\Gamma(n_\rho+1)}{\Gamma(n_\rho+K+3)}},$$

and b is oscillator length.

Expansion over oscillator basis reduces the set of integro-differential equations to the system of linear algebraic equations for expansion coefficients

$$\sum_{\tilde{n}_\rho,\tilde{c}}\left\{\left\langle n_\rho,c\left|\hat{H}\right|\tilde{n}_\rho,\tilde{c}\right\rangle - E\left\langle n_\rho,c\,|\,\tilde{n}_\rho,\tilde{c}\right\rangle\right\}C_{\tilde{n}_\rho,\tilde{c}}=0, \quad (8)$$

where multipole index c denotes channel of the hyperspherical basis $c=\{K,l_1,l_2;L\}$. This system is relevant for bound states and for continuous spectrum states. To obtain spectrum of bound states, one can use diagonalization procedure for the reduced set of the equations. However, to find wave functions and elements of the scattering S-matrix, one has to implement in (8) proper boundary conditions for expansion coefficients. These conditions were thoroughly discussed in Ref. [1].

3 Spectrum of resonance states in ^9Be and ^9B

To perform numerical calculations, we need to fix few parameters and select nucleon-nucleon potential. We start with selection of nucleon-nucleon potential. We exploit the Modified Hasegawa-Nagata potential (MHNP) [5, 6] to model nucleon-nucleon interaction. This is a semi-realstic potential and it was intensively used in numerous many-cluster systems, as it provides good description of the internal structure of clusters and interaction between clusters as well After NN potential was selected, we need to fix three input parameters: oscillator length b, number of channels or number of hyperspherical harmonics and number of hyper radial excitations. We restrict ourselves with a finite set of the hyperspherical harmonics, which is determined by maximal value of the hyperspherical momentum K_{min}. To describe the positive parity states we use allhyperspherical harmonics withthe hypermomentum $K \le K_{min} = 14$, the negative parity states are described by the hyperspherical harmonics with $K \le K_{min} = 13$.These amounts of the hyperspherical harmonics account for many different scenarios of three-cluster system decay. We also restrict ourselves with number of the hyperradial excitation $n_\rho \le 100$. This allows us to rich an asymptotic region, where all clusters are well separated and cluster-cluster interaction, induced by nucleon-nucleon potential, is negligible small.

Table 1 – Spectrum of bound and resonance states of ^9Be calculated with the MHNP

	Exp.		AM HHB, MHNP	
J^π	E (MeV ± keV)	Γ (MeV ± keV)	E (MeV)	Γ (MeV)
$3/2^-$	-1.5735		-1.5743	
$1/2^+$	0.111 ±7	0.217 ± 10	0.338	0.168
$5/2^-$	0.8559 ±1.3	0.00077 ±0.15	0.897	$2.363 \cdot 10^{-5}$
$1/2^-$	1.21 ±120	1.080 ± 110	2.866	1.597
$5/2^+$	1.476 ±9	0.282 ± 11	2.086	0.112
$3/2^+$	3.131 ±25	0.743 ± 55	4.062	1.224
$3/2_2^-$	4.02 ±100	1.33 ± 360	2.704	2.534
$7/2^-$	4.81 ±60	1.21 ± 230	4.766	0.404
$9/2^+$	5.19 ±60	1.33 ± 90	4.913	1.272
$5/2_2^-$			5.365	4.384
$7/2^+$			5.791	3.479

In present paper, the oscillator length b is selected to minimize the bound state energy of alpha particle, which is obtained with $b = 1.317$ fm. This allows us to describe correctly the internal structure of the alpha particle. If we take original form of the modified Hasegawa-Nagata potential, we obtain the overbound ground state in ^9Be and the bound state $3/2^-$ state in ^9B. The latter contradicts to experiments. The similar situation was observed for the Minnesota potential.To avoid this unphysical situation, we changed slightly parameters of the MHNP in order to reproduce bound state energy of ^9Be. Thus, by modifying the Majorana parameter, we obtain correct value of the binding energy of ^9Be. This is achieved with $m = 0.4389$, which can be compared to the original value $m = 0.4057$. With this value of the Majorana parameter, the spectrum of resonance states in ^9Be and ^9B is calculated.

Now we turn our attention to the spectrum of ^9Be and ^9B nuclei. Results of calculations with the MHNP is presented in Tables 1 and 2 where we compare our results with the experimental data [7]. Our calculations are in fairly good agreement with available experimental data. Energy and width of some resonance states are rather close to experimental data. For instance, parameters of $5/2^-$ and $9/2^+$ resonance states in ^9Be, and parameters $5/2^-$, $1/2^-$ and $5/2^+$ resonance states in ^9B.

That means that we found correct interaction between clusters in ^9B and ^9Be. In this paper as in the previous one [2], we use the same parameters of nucleon-nucleon interactions for all other J^π states. Comparing results of presvious and present calculations, we conclude that the modified Hasegawa-Nagata potential generates more correct cluster-cluster interaction for large set of the J^π states, thanthe Minnesota potential.We also conclude that spectrum of resonance states in ^9B and ^9Be strongly depends on peculiarities of nucleon-nucleon interaction.

Table 2 – Experimental and theoretical spectrum of resonance states of ^9B

	Exp.		AM HHB, MHNP	
J^π	E (MeV ± keV)	Γ (MeV ± keV)	E (MeV)	Γ (MeV)
$3/2^-$	0.277	0.00054 ±0.21	0.379	$1.076 \cdot 10^{-6}$
$1/2^+$	(1.9)	≃ 0.7	0.636	0.477
$5/2^-$	2.638 ±5	0.081 ± 5	2.805	0.018
$1/2^-$	3.11	3.130 ± 200	3.398	3.428
$5/2^+$	3.065 ±30	0.550 ± 40	3.670	0.415
$3/2^+$			4.367	3.876
$3/2_2^-$			3.420	3.361
$7/2^-$	7.25 ±60	2.0 ± 200	6.779	0.896
$9/2^+$			6.503	2.012
$5/2_2^-$			5.697	5.146
$7/2^+$			7.100	4.462

Now we concentrate our attention on the $1/2^+$ resonance states in ^9B and ^9Be. In Figures 1 and 2 we display phase shifts of $3 \Rightarrow 3$ scattering for the $1/2^+$ state in ^9B and ^9Be, respectively. These results are obtained with K_{max} = 14 and with the MHNP. With such value of K_{max}, 32 channels are involved in calculations and only three of them produces phase shifts which are not very small at energy region $0 \leq E \leq 5$ MeV. The phase shift connected with the channel $c = \{K = 0, l_1 = l_2 = L = 0\}$ of ^9Be shows resonance behavior at energies E = 0.338 MeV and E = 1.432 MeV.Second resonance state is also reflected in the second channel $c = \{K = 2, l_1 = l_2 = L = 0\}$ as a shadow resonance.

Phase shifts for $1/2^+$ state in ^9B also exhibit resonance states at two energies E = 0.636 MeV and E =2.875 MeV. As in case of ^9Be, $1/2^+$ resonance states in ^9B are connected with only one channel $c = \{K = l_1 = l_2 = L = 0\}$. Due to Coulomb interaction, resonance states in ^9B are shifted to higher energy range with respect to position of these resonance states in ^9Be.

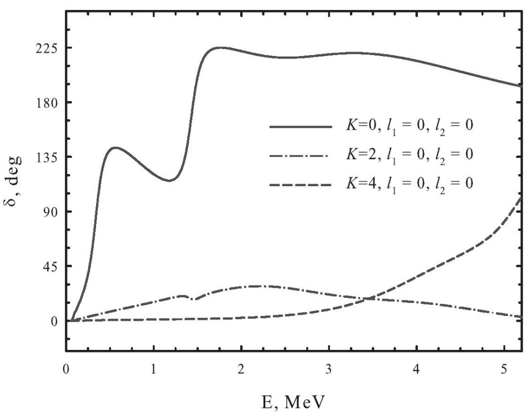

Figure 1 – Phase shifts for $3 \Rightarrow 3$ scattering in $1/2^+$ state in ^9Be

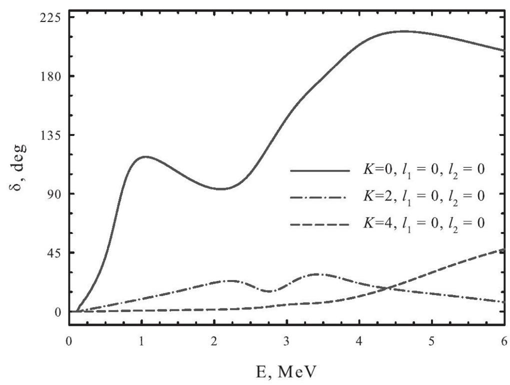

Figure 2 – Phase shifts for $1/2^+$ state in ^9B

To understand nature of $1/2^+$ and other resonance states in ^9B and ^9Be, we analyze wave functions. As was mentioned above wave function of three-cluster system is many-component and huge objects which is difficult to analyze. The simplest way for analyzing wave function of a resonance state is to study weights of oscillator shells. The weights are determined as follows

$$W_{sh} = W_{sh}\left(N_{sh}\right) = \sum_{n_\rho, c \in N_{sh}} \left| C_{n_\rho, c} \right|^2$$

It is important to note that oscillator wave functions with small values of N_{sh} describe very compact configurations of three-cluster system, when distance between interacting clusters is very small. Oscillator functions with large values of N_{sh} account for configuration of three-cluster system with large distance between all clusters and/or when one cluster is far away from two other clusters. In Fig. 3 we show the weight W_{sh} of different oscillator shell N_{sh} (N_{sh} = 0, 1, 2, ...) in wave function of resonance states. One can see that wave function of the $1/2^+$ resonance in ^9Be is similar to wave function of the resonance state in ^9B and both of them are represented by the oscillator shells with large values of N_{sh}. Figure 3 display behavior of wave function which is typical for low-energy wave functions. In asymptotic region these functions has an oscillatory behavior. Like in two-body case with sort-range interaction, the smaller energy, the larger is distance to the first node of wave function. In oscillator space we have approximately the same picture as in coordinate space. This is because there is simple relation between wave function in coordinate space and expansion coefficients in oscillator representation (see detail, for instance, in [1]).

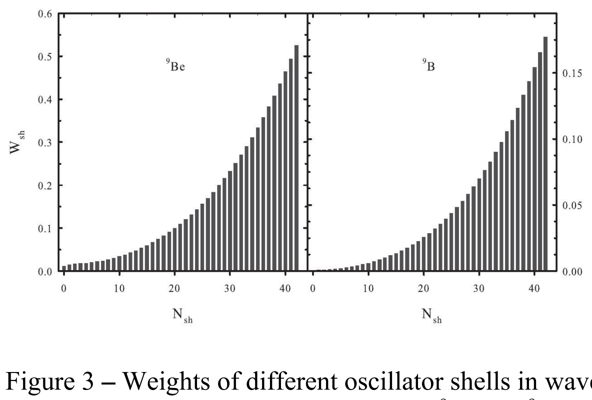

Figure 3 – Weights of different oscillator shells in wave functions of $1/2^+$ resonance states in ^9Be and ^9B.

By analyzing the total and partial widths, we determine the dominant decay channels of three-cluster resonance state. This analysis help us to shed some light on the nature of a resonance channel in many-channel system. It can be performed for two different trees of the Jacobi vectors, which were denoted as $n = {}^9Be$ and $^4He + {}^5He$ in Ref. [2]. The $1/2^+$ resonance state in ^9Be and ^9B has only dominant channel. In the first tree, the resonance prefer to decay into the channel, where the relative orbital momentum of two alpha particles and the orbital momentum of valence neutron (with respect to the center of mass of two alpha particles) equal zero. Partial width connected with that channel almost coincides with the total width. The same situation is observed in the second tree. There is also only one dominant channel with zero values of partial orbital momenta. The first orbital momentum represents relative motion of neutron around first alpha particle and the second one represents relative motion of second alpha particle with respect to the center of mass of the subsystem $\alpha + n$. These properties of the $1/2^+$ resonance states in ^9Be and ^9B are based on two important factors. First factor is the dominant role of the channel with the hypermomentum $K = 0$ in wave function of the resonance state. The second factor is connected with the essential properties of the hyperspherical harmonics with $K = 0$. With this value of hypermomentum, we have got only one hyperspherical harmonic which is independent on choice of the Jacobi vector tree.

Let us now consider the Hoyle analogue states in ^9Be. We recall that the Hoyle state is a very narrow resonance state in ^{12}C. It lies not far from the three-cluster threshold (E = 0.38 MeV) and has very small width Γ = 8.5 eV. This resonance state is created by collision of three alpha particles with total angular momentum and parity $J^\pi = 0^+$. As we see, the main features of the Hoyle resonance state that it is very long-lived resonance state (according to nuclear scale). If we look at Table 1, we find that ^9Be has two resonance states ($1/2^+$ and $5/2^-$) which lie close to the three-cluster threshold $\alpha + \alpha + n$. The $1/2^+$ resonance state is created by two values of the total orbital momentum L=0 and L=1. However, the resonance state is not narrow one, as ratio Γ / E is large $\Gamma / E \approx$ 0.5. Meanwhile, the $5/2^-$ resonance state is indeed narrow resonance state because width is small Γ = 23,6eV and besides ratio Γ / E is also very small: it equals $\Gamma / E \approx 2.63 \cdot 10^{-5}$ in our model and experimental ratio is $\Gamma / E \approx 9{,}0 \cdot 10^{-4}$. One can compare this ratio with the experimental ratio for the Hoyle state $\Gamma / E \approx 2{,}24 \cdot 10^{-7}$.

We believe that this resonance state is of the Hoyle-analogue state. This state has quite large half-life time, it could emit quadrupole gamma quanta and transit to the ground state of ^9Be. This is one of possible ways for synthesis of ^9Be. We assume, that in stars with large densities of alpha-particles and neutrons this is very plausible way of creating ^9Be nuclei.

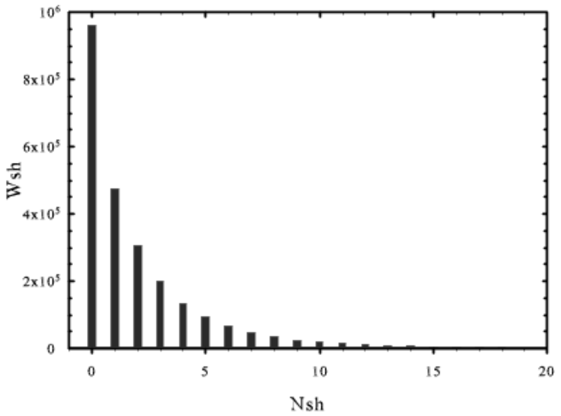

Figure 4 – Weights of different oscillator shells in wave function of the 5/2⁻ resonance state in ^9Be

In Figure 4 we demonstrate weight W_{sh} of different shells in wave function of the $5/2^-$ resonance state. It can be concluded from the Figure that the $5/2^-$ resonance state is very compact object, as it mainly represented by the oscillator shells with small number of N_{sh}. Besides, wave function of the resonance state has a very large amplitude in internal region ($W_{sh} \leq 10^6$). Such behavior of wave function of the $5/2^-$ resonance state in ^9Be is very similar to behavior of wave function of the Hoyle state in ^{12}C.

Three-cluster microscopic model was applied to study resonance states in mirror nuclei ^9Be and ^9B. The model make use of the hyperspherical harmonics to numerate channels of three cluster continuum and simplify of solving of the Schrödinger equation for many-particle and many-channel system. The modified Hasegawa-Nagata potential modelled nucleon-nucleon interaction. It was shown that the model with such NN interaction provides good description of parameters of resonance states. It was shown that $1/2^+$ states in ^9Be and ^9B are resonance states. Very narrow $5/2^-$ resonance state in ^9Be can be considered as the Hoyle-analogue state, we assume that this state is key resonance state for synthesis of ^9Be in a triple collision of alpha particles and neutron.

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References

[1] V. Vasilevsky. A. V. Nesterov. F. Arickx. and J. Broeckhove.Algebraic model for scattering in three-s-cluster systems. I. Theoretical background // – Phys. Rev.C. – 2001 – Vol. 63.–P. 034606.

[2] A. V. Nesterov. V. S. Vasilevsky. and T. P. Kovalenko.Nature of Resonance States in the ^9Be and ^9B Mirror Nuclei // Phys. Atom. Nucl.– 2014. – P. 77. –555–568.

[3] K. Wildermuth and Y. Tang.A unified theory of the nucleus // Braunschweig: Vieweg Verlag, 1977.

[4] R. Jibuti and N. Krupennikova. Method of Hyperspherical Functions in quantum mechanics of few body systems. (In Russian). // Tbilisi: Micniereba, 1984.

[5] A. Hasegawa and S. Nagata. Ground state of ^{6}Li // – Prog. Theor. Phys. – 1971. – Vol. 45. – P. – 1786–1807.

[6] F. Tanabe. A. Tohsaki. and R. Tamagaki. $\alpha\alpha$ scattering at intermediate energies. // Prog. Theor. Phys. – 1975. – Vol. 53. – P. 677–691.

[7] D. R. Tilley. J. H. Kelley. J. L. Godwin. D. J. Millener. J. E. Purcell. C. G. Sheu. and H. R. Weller. Energy levels of light nuclei A =8. 9. 10 // Nucl. Phys. A. – 2004. – Vol. 745. – P. 155–362.