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Lightest Kaonic Nuclear Clusters

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We present our study of kaonic three-body $\bar{K}NN$, $\bar{K}NN$ and $KK\bar{K}$ and four-body $\bar{K}NNN$, and $\bar{K}\bar{K}NN$ clusters within the framework of a potential model using the method of hyperspherical functions in momentum representation. To perform a numerical calculations for the bound state energy of the light kaonic system, we use a set of different potentials for the nucleon-nucleon and $\bar{K}N$ interactions, as well as for the kaon-kaon interaction. The calculations show that a quasibound state energy is not sensitive to the NN interaction, and it shows very strong dependence on the $\bar{K}N$ potential. We also compare our results with those obtained using different theoretical approaches. The theoretical discrepancies in the binding energy and width for the lightest kaonic system related to the different NN and $\bar{K}N$ interactions are addressed.

Key words: three-kaonic cluster, four-body kaonic cluster, hyperspherical harmonics
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1 Introduction

Nowadays, the study of exotic nuclear systems involving a $\bar{K}$ is an important topic in hadron physics, because the existence of kaonic nuclear states is related to kaon condensation and to physics of the core of neutron stars that by today's understanding are built up from exotic matter: pion and kaon condensates and quark matter [1, 2]. Kaonic nuclei carry important information concerning the $\bar{K}$ nucleon interaction in the nuclear medium. This information is very important in understanding kaon properties at finite density and in determining of the constraints on kaon condensation in high-density matter. The latter will allow one to adjust the methods developed in condensed matter physics for exciton and excitonic polariton condensates (see, for example, [3, 4]) to study the kaon condensation. The best way to understand the many body kaonic nuclear system is to study the simplest two-, three- and four-body clusters: $\bar{K}N$, $\bar{K}NN$ and $\bar{K}NNN$, as well as double kaonic clusters, when one nucleon in the three- or four-body kaonic cluster is replaced by the K – meson. The light kaonic clusters $\bar{K}NN$ and $\bar{K}NNN$ represent three- and four-body systems and theoretically can be treated within the framework of a few-body physics approaches. In the recent past much efforts have been focused on the calculations of quasibound state energies and widths for three- and four-body kaonic clusters. A variety of methods have been used in configuration and momentum spaces, to obtain eigenvalues for energy and width of quasibound states using diverse sets of $\bar{K}N$ and NN interactions. These include but are not limited by variational method approaches [5-15], the method of Faddeev equations in momentum and configuration spaces [16-28], Faddeev-Yakubovsky equations [26] and the method of hyperspherical harmonics in configuration and momentum spaces [29-28]. However, the predicted values for the binding energy and the width are in considerable disagreement. For example, for the K^-_{pp} cluster the predicted values for the binding energy and the width are 9–95 MeV and 20–110 MeV, respectively.

On the experimental side, several experiments have been performed to search for the kaonic clusters using various nuclear reactions starting from the first measurement reported by the FINUDA collaboration for the K^-_{pp} cluster [31] and including the most recent reports of J-PARC E15 and J-PARC E27 collaborations [32, 33] and HADES collaboration [34]. Recent HADES collaboration partial wave

analysis of the reaction $pp \to pK^+\Lambda$ at 3.5 GeV to search for the K^-_{pp} bound state shows that at a confidence level 95% such a cluster cannot contribute more than 2–12% to the total cross section with a $pK^+\Lambda$ final state [34]. However, there are important reports of K^-_{pp} search experiments done by DISTO collaboration and J-PARC E27 collaboration. They reported some signal at 100 MeV below the K^- and two protons threshold, which may be related to the kaonic cluster K^-_{pp}. J-PARC E27 collaboration has observed a K^-_{pp} – like structure in the $d(\pi^+, K^+)$ reaction at 1.69 GeV/c, while Ref. [35] reports an indication of a deeply bound K^-_{pp} state in the $pp \to p\Lambda K^+$ reaction at 2.85 GeV. The situation is still controversial and the existence, for example, of the K^-_{pp} quasibound state has not been established yet. Thus, the theoretical and experimental study of composite systems of K – mesons and nucleons is still a challenging issue in nuclear physics.

Below we present a study of the lightest kaonic nuclear clusters using the method of hyperspherical functions. We focus on three- and four-body nonrelativistic calculations within the framework of a potential model for the three- and four-body kaonic clusters using the method of hyperspherical harmonics (HH) in momentum representation. Calculations for a binding energy and width of the kaonic three- and four-body system are performed using different NN potentials and kaon-nucleon interactions, as well as kaon-kaon interactions. Such approach allows one to understand the key role of the kaon-nucleon interaction and the importance of nucleon-nucleon interaction in the formation of quasibound states of the kaonic three- and four-body systems.

2 Theoretical formalism

The Hamiltonians of the three and four nonrelativistic particles for the $\bar{K}NN$ and $\bar{K}NNN$ systems, respectively, read as

$$H_3 = \hat{T}_3 + V_{N_1N_2} + V_{\bar{K}N_1} + V_{\bar{K}N_2}, \quad (1)$$

$$H_4 = \hat{T}_4 + \sum_{1\le i<j\le 3} V_{N_iN_j} + \sum_{i=1}^{3} V_{\bar{K}N_i}, \quad (2)$$

where $\hat{T}_3$ and $\hat{T}_4$ are the operators of the kinetic energy for three- and four-particle system, respectively, $V_{N_iN_j}$ is the nucleon-nucleon potential and $V_{\bar{K}N_i}$ is a pairwise effective antikaon interaction with the nucleon. The effective interactions of the $\bar{K}N$, KN, $\bar{K}\bar{K}$ and $K\bar{K}$ two-body subsystems are discussed in detail in Refs. [5, 6, 10, 14, 36 – 39]. Below, we use two effective $\bar{K}N$ interactions that were derived in different ways. The effective $\bar{K}N$ interactions can be derived phenomenologically or constructed using the chiral SU(3) effective field theory, which identifies the Tomozawa-Weinberg terms as the main contribution to the low-energy $\bar{K}N$ interaction [37]. The potential for the description of the $\bar{K}N$ interaction was derived in Refs. [5, 10] phenomenologically, using $\bar{K}N$ scattering and kaonic hydrogen data and reproducing the $\Lambda(1405)$ resonance as a K^-_{pp} bound state at 1405 MeV, and the decay width of $\Lambda(1405)$ is also taken into account in this potential. We refer to this as the Akaishi-Yamazaki (AY) potential. The AY potential is energy independent. The other $\bar{K}N$ interaction given in Ref. [36] was derived based on the chiral unitary approach for the s – wave scattering amplitude with strangeness $S = -1$, and reproduces the total cross sections for the elastic and inelastic K^-p scattering, threshold branching ratios, and the $\pi\Sigma$ mass spectrum associated with the $\Lambda(1405)$. Hereafter we refer to this energy-dependent potential for the parametrization [40] as the HW potential. Both potentials are constructed in the coordinate space, are local, and can be written in the one-range Gaussian form as

$$V_{\bar{K}N}(r) = \sum_{I=0,1} U^I \exp\left[-(r/b)^2\right] P^I_{\bar{K}N}, \quad (3)$$

where r is the distance between the kaon and the nucleon, b is the range parameter and $P^I_{\bar{K}N}$ is the isospin projection operator . The values of the potential depth $U^{I=0}$ and $U^{I=1}$ for eachinteraction are given in Refs. [10] and [36] and the range parameter is chosen to be $b = 0.66$ fm for the AY potential and $b = 0.47$ fm for the HW potential.

To describe the $V_{N_iN_j}$ nucleon-nucleon interaction, we use several different NN potentials: the realistic Argonne V14 (AV14) and V18 (AV18) [41, 42] potentials, the semi-realistic Malfliet and Tjon MT-I-III (MT) [43] potential, the Tamagaki G3RS potential [44], which we hereafter refer to as the T potential, and potential [45], which we refer to as the M potential. Therefore, the use of different NN

potentials and $\bar{K}N$ interactions allows one to perform a validity test for the lightest kaonic clusters against various NN and $\bar{K}N$ interactions.

The binding energies and the wave functions of the three and four nonrelativistic particle can be obtained by solving the Schrödinger equation with the Hamiltonians (1) and (2), respectively. In our approach we use the hyperspherical harmonics method that represents a technique of solution of the Schrödinger equation to find the bound and scattering states for a few body system. The main idea of this method is the expansion of the wave function of the corresponding nuclear system in terms ofhyperspherical harmonics that are the eigenfunctions of the angular part of the Laplace operator in the six-dimensional space (three-body problem) or in the nine-dimensional space (four-body problem). The details of this method can be found in the monographs [46, 47, 48]. In our calculations we use the HH method in momentum representation [49, 48]. One starts from the Schrödinger equation for the three or four particles with the Hamiltonians (1) and (2), respectively, and rewrites this equation in the integral form in the momentum representation using the set of the Jacobi momenta $\mathbf{q}_i$ in $3(N-1)$ – dimensional momentum space. These momenta are the trees of Jacobi coordinates for three- or four-particle system $\mathbf{q}_i = \sqrt{\frac{m_{12\ldots i}m_{i+1}}{m_{12\ldots i+1}}}\left(\frac{1}{m_{12\ldots i}}\sum_{j=1}^{i} m_j\mathbf{q}_j - \mathbf{q}_{i+1}\right)$, $i = 1, 2, \ldots, N-1$, where m_j and $\mathbf{q}_j$ are the particles masses and momentum vectors conjugated to the position vectors $\mathbf{r}_j$ respectively, $m_{12\ldots i} = \sum_{j=1}^{i} m_j$ and N is the number of particles. After that, one introduces the set of the hyperspherical coordinates in the momentum space given by the hyperradius $\varkappa^2 = \sum_{i=1}^{N-1} q_i^2$ and the set of angles $\Omega_\varkappa$, which define the direction of the vector $\varkappa$ in $3(N-1)$ – dimensional momentum space, as well as the symmetrized hyperspherical harmonics in momentum representation $\Phi_\mu^\lambda(\Omega_\varkappa, \sigma, \tau)$ that are written as a sum of products of spin and isospin functions and hyperspherical harmonics [50]. Above, for the sake of simplicity, we denoted by λ the totality of quantum numbers on which the N – body hyperspherical harmonics depend and the integer μ is the global momentum in the $3(N-1)$ – dimensional configuration space, which is the analog of angular momentum in case of $N-2$. The HH are the eigenfunctions of the angular part of the $3(N-1)$ – dimensional Laplace operator in configuration space with eigenvalue $L_N(L_N+1)$, where $L_N = \mu + 3(N-2)/2$ and they are expressible in terms of spherical harmonics and Jacobi polynomials [46, 47, 48]. By expanding the wave function of N bound particles in terms of the symmetrized hyperspherical harmonics in momentum space

$$\Psi(\varkappa, \Omega_\varkappa) = \varkappa^{-\frac{3N-4}{2}} \sum_{\mu,\lambda} u_\mu^\lambda(\varkappa)\Phi_\mu^\lambda(\Omega_\varkappa, \sigma, \tau), \qquad (4)$$

and substituting Eq. (4) into the corresponding integral Schrödinger equation in the momentum representation, one obtains a system of coupled integral equations for the hyperradial functions $u_\mu^\lambda(\varkappa)$ for the system of three or four particles. The detailed description of the formalism for the K^-pp cluster can be found in Ref. [28]. Here we expand the wave function of three bound particles in terms of the symmetrized hyperspherical harmonics $\Phi_\mu^{l_p l_q L}(\Omega_\varkappa, \sigma, \tau)$ in momentum representation: $\Psi(\mathbf{p}, \mathbf{q}) = \sum_{\mu l_p l_q} u_\mu^{l_p l_q L}(\varkappa)\Phi_\mu^{l_p l_q L}(\Omega_\varkappa, \sigma, \tau)$, where μ is the grand angular momentum, L is the total orbital momentum, l_p and l_q are the angular momenta corresponding to the Jacobi momenta p and q that are conjugated to the standard Jacobi coordinates for three particles, $\varkappa$ is the hyperradius in the six dimensional momentum space, and $\Omega_\varkappa$ is the set of fiveangles which define the direction of the vector $\varkappa$. The functions $\Phi_\mu^{l_p l_q L}(\Omega_\varkappa, \sigma, \tau)$ are written as a sum of products of spin and isospin functions and HH, using the Raynal-Révai coefficients [51]. For the system K^-pp the wave function is antisymmetrized with respect to two protons, while for the K^-K^-p system it is symmetrized with respect to two antikaons. For the hyperradial functions $u_\mu^{l_q l_p L}(\varkappa)$ we obtain the coupled integral equations. By solving the coupled integral equations one can find the hyperradial functions $u_\mu^{l_q l_p L}(\varkappa)$ for a given L and the binding energies for the K^-pp and K^-K^-p systems. For the system $\bar{K}NNN$ the wave function is antisymmetrized with respect to three nucleons, while for the K^-K^-pp system it is symmetrized with respect to two antikaons and antisymmetrized with respect to two protons. The hyperradial functions

$u_{\mu}^{\lambda}(\varkappa)$ for four-body systems can be found by solving the coupled integral equations and use them to construct the corresponding wave functions (4). To solve the coupled integral equations for the hyperradial functions $u_{\mu}^{\lambda}(\varkappa)$ for the system of three or four particles obtained from the corresponding Schrödinger equations, we include only the real part of the $\bar{K}N$ $K\bar{K}$ and $\bar{K}\bar{K}$ interactions, quite in the same way as the earlier variational studies [13, 14, 29]. Using the wave function, the width of the bound state can be evaluated in a perturbative way from the imaginary part of the $\bar{K}N$ interaction as $\Gamma = -2\left\langle \Psi \left| \mathrm{Im}\left(V_{\bar{K}N}(r_{12}) + V_{\bar{K}N}(r_{13}) \right) \right| \Psi \right\rangle$ for $K^{-}pp$ and $K^{-}K^{-}p$ clusters. As it is stated in review [52], as well as demonstrated in the recent calculations of the width for the $K^{-}pp$ system [54] using a coupled-channel complex scaling method with Feshbach projection, this is a reasonable approximation. For an approximate evaluation of the width the imaginary part of the complex potential has often been treated perturbatively in the early variational studies [13, 14, 29] and by many authors, see for example [8, 9, 12, 13, 14, 15, 29, 30, 39]. In the same way the width of the bound state for $KK\bar{K}$ system is evaluated from the imaginary part of the $\bar{K}K$ interactions as $\Gamma = -2\left\langle \Psi \left| \mathrm{Im}\left(V_{\bar{K}K}(r_{12}) + V_{\bar{K}K}(r_{23}) \right) \right| \Psi \right\rangle$ and the widths for the $\bar{K}NNN$ clusters are evaluated through the expression $\Gamma = -2\left\langle \Psi \left| \mathrm{Im} v_{\bar{K}N} \right| \Psi \right\rangle$, where $v_{\bar{K}N}$ sums over all pairwise $\bar{K}N$ interactions.

In calculations with the energy dependent HW potential we follow Ref. [14] and use a " corrected" energy dependent complex potential, where the strength for each channel is determined so as to reproduce the $\bar{K}N$ scattering amplitude predicted in Ref. [40] and is parametrized by polynomial in terms of the $\bar{K}N$ energy. Also, to determine the $\bar{K}N$ energy in the $K^{-}pp$ system, the authors of Ref. [14] examined two ansatz, "Type I" and "Type II", which are given as Eqs. (20) and (21) in Ref. [14], respectively. In the current study is employed the "Type II" ansatz.

In the following Section we present results for a single-channel calculation using effective $\bar{K}N$, $\bar{K}\bar{K}$ and $\bar{K}\bar{K}$ interactions.

3 Results of numerical calculations and discussion

3.1 $K^{-}pp$ cluster

Let's start with the results of our calculations of the $K^{-}pp$ cluster recently reported in Ref. [28]. Results of these calculations for the $K^{-}pp$ cluster are presented in Table 1. For the calculations of the binding energy and the width with the method of HH we use as input MT, T, and AV14 potentials for the NN interaction, while for the $\bar{K}N$ interaction we use the energy-dependent effective HW and the phenomenological AY potentials. Such an approach allowed us to examine how the $K^{-}pp$ cluster's structure depends on different choices of the $\bar{K}N$ interactions for the same NN potential, as well as to investigate its dependence on different choices of the NN interaction for the same $\bar{K}N$ interaction, and to understand the sensitivity of the system to the input interactions.The analysis of the calculations presented in Table 1 show that the AY potential as the $\bar{K}N$ interaction input falls into the 46-47 MeV range forthe binding energyof the $K^{-}pp$ cluster, while the chiral HW $\bar{K}N$ potential gives about 17-21 MeV for the binding energy. Thus, the values for the binding energy for the $K^{-}pp$ system obtained for the different NN potentials are in reasonable agreement, and the ground state energy is not very sensitive to the NN interaction. However, there is a very strong dependence on the antikaon-nucleon interaction. When we employ the effective energy-dependent chiral theory based HW potential for the $\bar{K}N$ interaction and different NN interactions, as inputs, we predict a weakly bound $K^{-}pp$ cluster. This is similar to Ref. [14], where the authors employed several versions of energy-dependent effective $\bar{K}N$ interactions derived from chiral SU(3) dynamics together with the realistic AV18 NN potential. Our calculations also confirm results reported in earlier studies [13, 21, 29] employing the same type of $\bar{K}N$ interaction. The energy of the bound state, as well as the width calculated for the AY potential are more than twice as big as those obtained for the energy-dependent chiral $\bar{K}N$ HW potential. Therefore, the highest binding energies are obtained for the phenomenological AY potential. Let's compare our results with those obtained with different variational approaches. Our result for the binding energy is in good agreement with the result from Ref. [10], where the binding energy for the $K^{-}pp$ cluster was calculated

by employing the AY potential as the $\bar{K}N$ interaction and T potential as the NN interaction. However, the decay width seems rather different among two studies: in the present study the width is 74.5 MeV, while that obtained in Ref. [10] is 61 MeV. This should be related to the different behavior of the waves functions obtained using the variational approach and the method of hyperspherical functions. Recently it was reported that "resonance and coupled-channel problem are key ingredients in the theoretical study of the K^-pp" [54]. Those authors employ a coupled-channel Complex Scaling Method combined with the Feshbach method since this approach can simultaneously treat these two ingredients. Interestingly enough, their calculations [54, 53] for the binding energy and width are consistent with our results obtained within the single channel potential model. The comparison of our calculations with results obtained using the HH method in configuration space [29] and differential Faddeev equations [28] also are in reasonable agreement. This is a good indication that the binding energy does not depend significantly on the method of calculation.

3.2 K^-K^-p cluster

Three-body problem with two mesons and one baryon have received considerable attention in the recent literature [38, 39, 55, 56]. The baryonic systems $\bar{K}\bar{K}N$ and $\bar{K}KN$ with two kaons were investigated in Refs. [38, 39, 57]. We study a possible bound state of the K^-K^-p cluster with $S = -2$, $I = 1/2$, $J^+ = 1/2^+$ using the effective s – wave AY and HW potentials assuming that this state is formed due to the strong K^-p attraction.

The strength of the $s-$wave $\bar{K}\bar{K}$ interaction for the isospin $I = 0$ is zero due to Bose statistics, and we consider a weak repulsion for the isospin $I = 1$ that reproduces the scattering lengths $a_{K^+K^+} = -0.14$ fm for the range parameter $b = 0.66$ fm (AY potential) and $b = 0.47$ fm (HW potential). The results of calculations for the binding energies for the K^-p and K^-K^-p, the bound K^-K^-p state without K^-K^- interaction, and the root-mean-square radius of the $\bar{K}$ distribution are presented in Table 2. For the AY potential, the K^-K^-p system is still bound even with a much stronger $\bar{K}\bar{K}$ repulsion, while for the HW potential there is the bound state with the energy 0.01 MeV relative to the $K^-p + K^-$ threshold. Thus, although the $\bar{K}N$ with $I = 1$ is attractive, the attraction is not strong enough to overcome the $\bar{K}\bar{K}$ repulsion. For the width within the method of HH we obtain 58.6 MeV and 41.6 MeV with the AY and HW potentials, respectively. Our results for the binding energy of thesystem obtained by the method of HH are in reasonable agreement with calculations obtained using a variational method [39] and the Faddeev calculations [26].

Table 1 – The binding energy B and width Γ for the K^-pp system calculated in the framework of the method of HH in the momentum representation for different interactions. NN potentials: AV14 [41], MT [43] and T [44]. $\bar{K}N$ interactions: AY [10] and HW [36]. E_{K^-p} is two-body energy in the K^-pp cluster.

	AV14+AY	MT+AY	T+AY	AV14+HW	MT+HW	T+HW
B, Mev	46.2	46.5	46.3	17.2	20.5	20.6
Γ, MeV	66.7	84.3	74.5	44.3	48.1	49.5
E_{K-p}, Mev	29.9			10.9		

Table 2 – The bound state energies of K^-p (E_2) and K^-K^-p (B) systems, and the root-mean-square radius of thedistribution.is the binding energy measured from the two-body threshold

K^-K^-	K^-p	$< r^2 >^{1/2}, fm$	E_2, MeV	B, MeV	$\Delta E, MeV$
AY	AY	1.36	30.0	31.7	1.7
$V_{K^-K^-} = 0$	AY			35.3	5.3
HW	HW	1.96	11.42	11.43	0.01
$V_{K^-K^-} = 0$	HW			12.21	0.79

3.3 $KK\bar{K}$ system

Recently, there has been increased interest in few-body systems constituted by two or more kaons. Particularly noteworthy is the possibility of formation of the quasibound states in a $KK\bar{K}$ system. We study the $KK\bar{K}$ system using a nonrelativistic potential model in the framework of the method of HH in momentum representation and consider the $KK\bar{K}$ system as three interacting kaons. Once the two-body interactions for the $K\bar{K}$ and KK subsystems are determined one can determine the wave function of the $KK\bar{K}$ system by solving the Schrödinger equation for the Hamiltonian $H = \hat{T}_3 + V_{\bar{K}K}(r_{12}) + \quad + V_{KK}(r_{13}) + V_{\bar{K}K}(r_{23})$ where the potential energy is the sum of the effective $K\bar{K}$ and KK interactions that are the functions of the interparticle distances r_{ij}. For the description of the effective kaon-kaon interactions we use the local potentials from Refs. [39] and [38] that can be written in one-range Gaussian form (3). The set of values of the potential depth U_A^I for eachinteraction is given in Refs. [39, 38] and the range parameteris chosen to be the same forandinteractions. We choose two optimized values for the range parameter: b = 0.66 fm (set A) and b = 0.47 fm (set B). The strength of strongly attractive s – wave $K\bar{K}$ interactions was assumed to be the same for the isospin I = 0 and isospin I = 1, while the strength of the s – wave $K\bar{K}$ interaction for the isospin I = 0 is $U_{KK}^{I=0}$ due to Bose statistics and we consider a weak repulsion for the isospin I = 1. In Ref. [39] the $K\bar{K}$ interaction is derived under the assumption thatforms the quasibound states f_0 (980) and a_0 (980) in I = 0 and I = 1 channel, respectively, and it reproduces the masses and widths of these resonances. The strength of the repulsive KK interaction inwas fixed to reproduce a lattice QCD calculation [58] for the scattering length $a_{K^+K^+} = -0.14$ fm, and a weaker repulsion that corresponds to the scattering length $a_{K^+K^+} = -0.10$ fm. Results of calculations for the set of potentials A when the KK interaction reproduces the scattering lengths $a_{K^+K^+} = -0.14$ fm and $a_{K^+K^+} = -0.10$ fm are denoted as A1 and A2, respectively. Correspondingly, the set of potentials Bthat reproduces those different scattering lengths hereafter we refer as B1 and B2.

Table 3 – Results of calculations of different characteristics of the $KK\bar{K}$ system

	Fadeev[59]	A1[59]	A1	A2	B1	B2	Separable AMY [60] potential
Mass, Mev	1420	1467	1469.4	1468.2	1464.1	1463.8	1463.4
$\Gamma/_2$, MeV	25	55	42	41.1	48.4	49.1	–
B, Mev		21	18.6	19.8	23.9	24.2	24.6
$\sqrt{< r^2 >}$, Mev		1.6	1.72	1.65	1.61	1.56	1.52
KK distance, fm		2.8	3.2	2.92	2.72	2.70	2.68
$(KK) - \bar{K}$, fm		1.7	1.78	1.68	1.66	1.62	1.60
$K\bar{K}$ distance, fm		1.6	1.68	1.65	1.64	1.58	1.55
$(K\bar{K}) - \bar{K}$ distance, fm		2.6	2.9	2.86	2.55	2.50	2.47

The solution of a system of coupled integral equations for the hyperradial functions allows us to construct the wave functionΨ for the $KK\bar{K}$ system and to determine the binding energy B. A reasonable convergence for the ground state energy is reached for the grand angular momentum $\mu_{\max} = 10$ and we limit our considerations to this value. The results of our calculations for the binding energy and the width for the $KK\bar{K}$ system along with the results obtained with a coupled-channel approach based on the solution of the Faddeev equations in momentum representation and the variational method [59] are presented in Table 3. The total mass the $KK\bar{K}$ system ranges from 1463.4 to 1469.4 MeV when we consider the sameK meson mass $m_K = 496$ MeV as in Ref. [59]. The width falls within the 41 - 49 MeV range for all sets of theandinteractions. The quasibound state for the $KK\bar{K}$ with spin-parity0$^-$and

total isospin 1/2 is found to be below the three-kaon threshold. The comparison of our results with the results for the binding energy 21 MeV (the mass is 1467 MeV) and the width 110 MeV obtained with the variational method in Ref. [59] shows that while the binding energy found within the HH and variational calculations are close enough, the percent differences of the results for the width is less than 26%. A reason of this difference is related to the different behavior of the wave function of the $KK\bar{K}$ system obtained within the variational method and the method of HH. The wave function within the method of HH is obtained using the criterion of conversion of the binding energy with the accuracy about 0.2 MeV and consideration of the next terms with $\mu > 10$ in the expansions (4) only very slightly changes the binding energy. However, the width that is calculated using the perturbative approach more sensitive to the wave function and does not converge so fast as a binding energy. This leads to the different overlapping of the imaginary part of the $V_{\bar{K}K}(r)$ potential. The difference between the HH and Faddeev calculations [59] is understandable because in Ref. [59] thesystem is studied with a coupled-channel approach based on solving the Faddeev equations considering the $KK\bar{K}$, $K_{\pi\pi}$, and $K_{\pi\eta}$ channels and using as input two-body matrices that generate f_0 (980) and a_0 (980)resonances, while in the present calculations we use a single-channel three-body potential model. In addition our calculation is carried out in non-relativistic approach, whereas the Faddeev calculation is done in semi-relativistic approach using two-body amplitudes that are calculated by solving a relativistically covariant Bethe-Salpeter equation in a coupled-channel approach and using the on-shell factorization method. Such a difference could make large discrepancy in the obtained results.

We also perform calculations for the $KK\bar{K}$ system using s–wave two-body separable potentials with Yamaguchi form factors from Ref. [60] that also used in Faddeev and Faddeev-Yakubovsky calculations [26] for K^-K^-p theand K^-K^-pp kaonic clusters. The corresponding results are presented in the last column of Table 3 and are very close to the results obtained using the effective local kaon-kaon interactions for the set B. Thus, our calculations within three body nonrelativistic potential model predict a quasibound state for the $KK\bar{K}$ system with mass around 1460 MeV that can be associated with the K(1460) resonance. Our results support the conclusion obtained through the variational calculations that K(1460) could be considered as a dynamically generated resonance.

3.4 $\bar{K}NNN$ clusters

Recently Faddeev-Yakubovsky calculations [26] were made for the four-particle K^-ppn and K^-K^-pp kaonic clusters, where the quasibound states were treated as bound states by employing real s–wave two-body separable potential models for the $\bar{K}\bar{K}$ and $\bar{K}$ the nucleon interactions as well as for theinteraction. Fully four-body nonrelativistic realistic calculations of $\bar{K}NNN$ and $\bar{K}\bar{K}NN$ quasibound states within the method of HH in configuration space, using realistic NNpotentials and subthreshold energy dependent chiral $\bar{K}N$ interactions, were presented in Ref. [29]. Giving that below we present the results of our calculations for the $\bar{K}NNN$ and $\bar{K}\bar{K}NN$ quasibound states in the framework of method of HH in momentum representation using AV18 [42] and M [45]potentials and AY and HW $\bar{K}N$ interactions as inputs. To find the binding energies with above mentioned set of potentials, we solve a system of coupled integral equations for the hyperradial functions $u_\mu^\lambda(\chi)$. In the calculations we limit our consideration with the value μ_{max} = 10 getting a reasonable convergence for the binding energy. In Table 4 we present our results for $\bar{K}NNN$ cluster that we compare with those obtained via different methods. The results of our calculations for the energy and the width show dependence on theNN potentials and on the $\bar{K}N$ interactions. However, this dependence is dramatically different: for the same $\bar{K}N$ interaction and different NN potentials the ground state energy and the width change by about 3-15 %, while for the sameNN potential and differentinteraction the energy changes by a factor of more than 3 and the width changes by more than twice.

Table 4 – The binding energyand widthfor thesystem calculated in the framework of the method of HH in the momentum representation for different interactions with results from Refs. [9], [26] and [29]. The parity includes the eigen parity of antikaon.

J^π	T		AV18+AY	M+AY	AV18+HW	M+HW	[29]	[26]	[9]
$K^-pnn\ \frac{1}{2}^{-1}$	0	B, Mev	92.1	97.9	28.6	28.9	29.3	69	110.3
		Γ, Mev	83.4	84.1	30.3	30.8	32.9		21.2
$K^-pnn\ \frac{1}{2}^{-1}$	1	B, Mev	64.6	66.7	17.2	18.7	18.5		
		Γ, Mev	74.2	80.4	27.1	31.4	31.0		
$K^-ppp\ \frac{1}{2}^{-1}$	1	B, Mev	101.9	107.6	25.8	28.1			96.7
		Γ, Mev	87.9	89.8	28.1	31.2			12.5

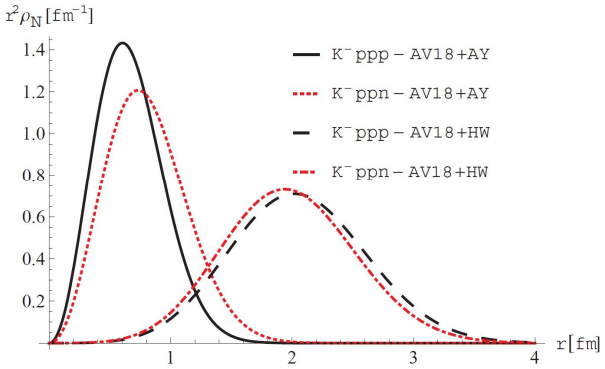

Figure 1 – Nucleon density distributions for and clusters

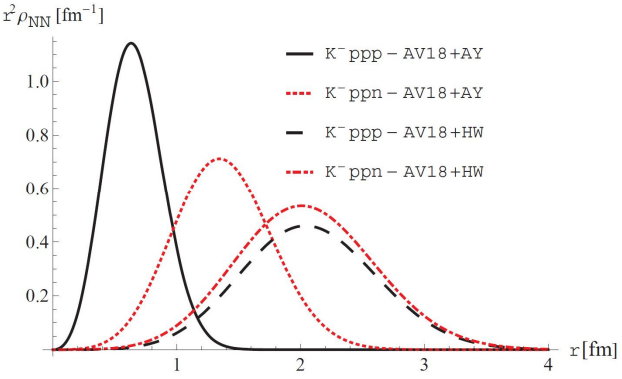

Figure 2 – Dependence of the two-nucleon density distributions in and clusters on the internucleon distance

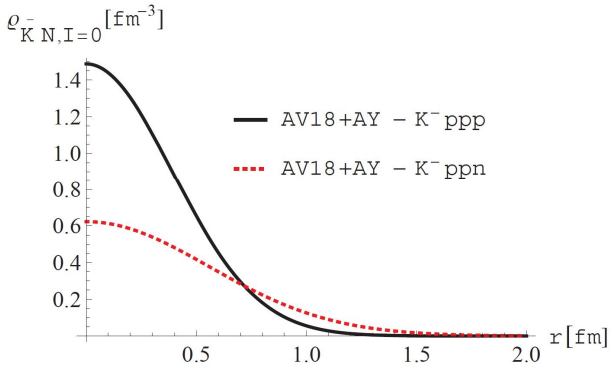

Figure 3 – Dependence of thedensity with isospininandclusters onrelative distance

For the comparison let's mention that the authors of Ref. [29] obtained 29.3 MeV and 32.9 MeV, and 18.5 MeV and 31.0 MeV for the binding energy and the width ofthe K^-ppn and K^-pnn clusters, respectively, while calculation within the Faddeev-Yakubovsky equations [26] with separable potential models for the $\bar{K}$ –nucleon and the nucleon-nucleon interactions leads to very deep ground state energyMeV forsystem. The comparison of our results for the K^-ppn and K^-pnn clusters obtained for AV18NN interaction and HW $\bar{K}N$ interaction with calculations [29] within the variational HH method for the AV14 NNinteraction and shallow chiral $\bar{K}N$ interaction shows a reasonable agreement. The predictions [9] for the binding energy and the width for the kaonic clusters studied

based on a framework of antisymmetrized molecular dynamics and employing adopted AY [5] potential as a bare $\bar{K}N$ interaction and the type of T [44] potential as a bare NNinteraction are presented in the last column of Table 4. In our calculations we use the same parametrization of the AY and HW potentials as in Section III A. As is seen from Table IV there is a reasonable agreement for the widths for the K^-ppn and K^-pnn clusters among the present study and study [29] in case of HW potential. However, the widths of the K^-ppn and K^-pnn systems are largely different among our study and an earlier study [9] in the case of AY potential. To understand these discrepancies and compare our results with Ref. [9], we performed the calculations for the $\bar{K}N$ interaction $V_{\bar{K}N}^{I=0}(r) = -(593 + i\omega_0)\exp[-(r/0.66)^2]$ and use for the imaginary part of the potential the different values of $\omega_0 = 20$, 40, 83 MeV, respectively [5, 9]. The results are the following: K^-ppp = (101.9, 19.8) MeV, (101.9, 38.7) MeV, (101.9, 87.9) MeV; $K^-ppn, (B,\Gamma)$ =(92.1, 18.7) MeV, (92.1, 37.4) MeV, (92.1, 83.4) MeV; $K^-pnn, (B,\Gamma)$=(64.6, 16.3) MeV, (64.6, 32.6) MeV, (64.6, 74.2) MeV, that are in agreement with the corresponding calculations from Refs. [5, 9] that lead to narrow widths only for small values of ω_0 . If the binding energy is large and lies below the threshold of the main decay channel $\pi\sum$, as a result we have the width of the quasi-stable discrete bound state less than the binding energy and complex part of the $\bar{K}N$ potential should be small. The value $\omega_0 \sim 20$ MeV can reproduce the results of the width from Refs. [5, 6, 7, 8, 9]. In our approach we can reproduce the narrow width reported in Ref. [9] only for the small value of the the imaginary part of the AY potential. Interestingly enough, our calculations for the AY interaction indicate that for the system $\bar{K}NNN$ the cluster K^-ppp is more deeply bound K^-ppn than thethat contradicts to the results [9]. In a shell-model picture, one of three protons in the K^-ppp should be raised up to 0p orbit due to Pauli principle, while all nucleons in K^-ppn occupy the 0s orbit. So, the naïve expectation is that the K^-ppn is lower than the K^-ppp energetically. From the other side the larger number of the strongly attractive K^-p pairs in the $(\frac{3}{2}^+,1)$ state than in $(\frac{1}{2}^-,0)$ state may cause a lowering of theT=1 state, even below the T = 0, although the third proton in the T = 1 state should be flipped up to the orbital $(0p_{3/2})$. The final picture depends on the strength of $\bar{K}N$ interaction. The attractive AY interaction is much stronger than the effective HW interaction. The strength of $\bar{K}N$ interaction plays an important role that may lead to the nuclear compression. Following Ref. [61] we calculated a nucleon density distribution $\varsigma_N(r)$ averaged over angular dependence multiplied by r^2, wherer is the distance of the nucleon from the center of mass for K^-ppp, or K^-ppn clusters, shown in Fig. 1. For K^-ppp cluster one can observe a significant spatial shrinkage when the AY $\bar{K}N$ interaction is used. Since the $\bar{K}N$ potential is much more attractive in the I = 0 channel than in the I=1channel, different distribution of protons and neutrons is expected in kaonic clusters. Such results have already been reported in the early study within a new framework of the antisymmetrized molecular dynamics method in Refs. [8] and [9]. Probably in a $\bar{K}$ – nuclear system is preferable a structure where the proton distribution differs from the neutron distribution, such a structure dynamically produced in the hyperspherical function treatment. Particularly, the composition of the K^-ppp wave function within our formalism is the following:

$$[K^-pp(T=1/2,J=0)+p]^{3/2^+,1}, -93\%,$$
$$[K^-p(T=0,J=1/2)+pp]^{3/2^+,1}, -6.4\%,$$
$$[K^-+(ppp)(T=3/2,J=3/2)+p]^{3/2^+,1}, -0.6\%.$$

Our calculations show that the dominant contribution into the total wave function of the K^-ppp system is the $[K^-pp(T=1/2,J=0)+p]^{3/2^+,1}$ configuration. While the K^-p interaction is much stronger than the K^-n, the protons preferably allocate near the K^- and their kinetic energy increase. However, the total energy decreases due to the strongly attractive AY K^-p interaction. As a result the K^-ppp binding energy of thebecomes larger than that for the K^-ppn. In Table 5 are presented the kinetic energy per nucleon for a proton and a neutron in $\bar{K}NNN$, as well as the expectation value of the $\bar{K}N$ interaction. To estimate the expectation values of the kinetic energy and of the $\bar{K}N$ potential energy per nucleon the particle numbers are counted following Ref. [9] using Clebsch-Gordan coefficients. The particular numbers of protons and neutrons are 2.67 and 0.33, respectively, for K^-ppp and both are equal 1.5 for K^-ppn. Analysis of Table 5 and all above notes gives a possible explanation why the K^-ppp cluster is more deeply bound than the K^-ppn when the strongly attractive AY potential [10] is used, while

when the input is the HW potential we have an opposite picture. To reveal the characteristic structure of K^-ppp and enhancing the difference between the K^-ppp and K^-ppn clusters we calculate density distributions of the NN and $\bar{K}N$ pairs as functions of the respective nucleon-nucleon and antikaon-nucleon distances using Eq. (32) from Ref. [14]. In Fig. 2 is shown two-nucleon density distribution $\varsigma_{NN}(r)$ multiplied by r_2 in the K^-ppp and K^-ppn clusters for the AY and HW potentials. The two-nucleon distribution shows the pronounced maximum at the short-distances 0.63 fm and 1.34 fm for the K^-ppp and K^-ppn clusters, respectively, in the case of strong AY interaction. For relatively shallowly bound by the HW potential K^-ppp and K^-ppn clusters the maximum is pronounced at 2.02 fm and 2.05 fm, respectively, that reflects relatively weak binding of the systems. One can also understand a reason for the deeper binding of K^-ppp and K^-ppn thanin case of AY potential by analyzing thedensity distribution. Because the $\bar{K}N$ potential in isospin-zero channel plays a key role in the deep binding of $\bar{K}$ clusters, we calculate the projected density distributions for $\bar{K}N$ pairs with isospin $I = 0$ in the K^-ppp and K^-ppn clusters. The results of calculations of the normalized projected density $\varsigma_{\bar{K}N,I=0}(r)$ are shown in Fig. 3. As is seen in Fig. 3 the $\bar{K}N$ density distribution with isospin $I = 0$ has its maximum at zero distance between the antikaon and each nucleon that reflects the strong $\bar{K}N$ attraction in the $I = 0$ channel. The comparison of the $\bar{K}N$ density distribution shows that one is bigger for the K^-ppp cluster compare to that for the K^-ppn. From the projected density distributions for the $\bar{K}N$ pairs with isospin $I = 0$ configuration calculated mean-square distances $R_{\bar{K}N}$ are 1.45 fm and 2.04 fm for the K^-ppp and K^-ppn clusters, respectively. The later facts reflect relatively strong binding of the K^-ppp system. Thus, the K^-ppp is compacter than the K^-ppn cluster.

Based on the results of our calculations, we can conclude that the pairwise $\bar{K}N$ interaction plays a major role in the formation of the kaonic bound state and the effective $\bar{K}N$ interaction based on chiral SU(3) dynamics [36] leads to a relatively modest binding for the K^-ppm, K^-pmm and K^-ppp clusters. Our results confirm the calculations [29].

Table 5 – Nucleon energy inandclusters. Expectation values of the kinetic energyand of theinteraction per nucleon calculated in the framework of the method of HH in the momentum representation for the AY and AV18 interactions

		$< T >$MeV/N	$< \bar{K}N >$ MeV/N
K^-ppp	Proton	78.1	-195.2
	Neutron	49.3	-29.1
K^-ppn	Proton	72.4	160
	Neutron	55	38.2

$\bar{K}\bar{K}NN$ cluster

A decade ago in Ref. [7] a deeply bound double K^-K^-pp cluster was predicted to be deeply bound with binding energy of 117 MeV and width 35 MeV. Barnea, Gal and Liverts [29] perform a variational HH calculation in configuration space for the K^-K^-pp system based on the shallow chiral $\bar{K}N$ interaction model with the self-consistent energy dependence taken into account and obtained very shallow bound states with a binding energy that is substantially smaller than earlier prediction [7]. When in our calculation the HW potential is used the similar result to the early study [29] is obtained. In our calculations with the AY interaction, as in Section III D, we employ the potential $V_{\bar{K}N}^{I=0}(r) = -(593 + i\omega_0)\exp[-(r/0.66)^2]$ using for the imaginary part of the potential the values ω_0 = 20, 40, 83 MeV, respectively. In Table VI are presented results when ω_0 = 83 MeV. The corresponding values for the width are 72.4 MeV and 73.7 MeV for AV18 and M potentials, respectively, that are close enough to the widths obtained using the HW interaction. However, the binding energy is almost three times bigger. When $\omega_0 = 20$MeV and 40 MeV, respectively, Γ = 17.9 MeV and Γ = 35.8 MeV for the AV18 potential, and Γ = 18.5 MeV and Γ = 37.1 MeV for M potential. Thus, our results with the AY $\bar{K}N$ interaction are close to the earlier prediction [7] when ω_0 = 40 MeV and the binding energy is in a reasonable agreement with recent Faddeev-Yakubovsky calculations [26].

Table 6 – The binding energyand widthfor thesystem calculated in the framework of the method of HH in the momentum representation for different interactions with results from Refs. [7], [26] and [29].

		AV18+AY	M+AY	AV18+HW	M+HW	[29]	[26]	[7]
K^-K^-pp	B, Mev	91.6	92.7	31.5	31.9	32.1	93	117
	Γ, Mev	72.4	73.7	78.1	79.2	80.5		35

4 Conclusions

Within the framework of a potential model for the kaonic clusters K^-pp, K^-K^-p, $KK\bar{K}$, $\bar{K}NNN$, and K^-K^-pp we perform nonrelativistic three- and four-body calculations using the method of hyperspherical harmonics in the momentum representation. We examine how the binding energy and width of the K^-pp cluster depends on different choices of the $\bar{K}N$ and NN interactions. Our consideration includes the realistic Argonne V14 [41], the semi-realistic MT [43] and T [44] potentials as inputs for the NN interaction and we employ the phenomenological AY potential and HW potential constructed based on chiral SU(3) dynamics, as inputs for the $\bar{K}N$ interaction. For all types of considered NN interactions, our calculations predict deeply bound states for the AY $\bar{K}N$ interaction and a relatively shallowly boundcluster for the effective chiralinteraction. Moreover, the K^-pp cluster is the most strongly quasibound three-body system. The results of our calculations show that the binding energy of the K^-pp system depends entirely on the ansatz for the $\bar{K}N$ interaction and substantially changes when we use the AY and HW $\bar{K}N$ interaction. In regard to the sensitivity of the binding energy to the details of the NN potentials, Ref. [14] found that when the K^-pp system is weakly or deeply bound, the dependence on different types of NN interactions is weak. In fact, our study confirms this conclusion using in calculations Argonne V14 [41], the semi-realistic MT [43] and T [44] NN potentials.

The strong AY $\bar{K}N$ interaction is responsible for the formation of the K^-K^-p system and this cluster is still bound even with a much stronger $\bar{K}\bar{K}$ repulsion, while the HW potential leads to the bound state with energy of only 0.01 MeV relative to the $K^-p + K^-$ threshold. The mass (binding energy) of the $KK\bar{K}$ system slightly depends on the sets of parameters that determine $K\bar{K}$ and KK interactions and the width falls into the 41-49 MeV range for all sets of these parameters. There is reasonable agreement between these results, the mass obtained using separable AMY interactions [60] and the variational calculation [59]. Our results for the $KK\bar{K}$ system support the conclusion that K (1460) could be considered as a dynamically generated resonance.

Based on the results of our calculations for four-particle kaonic systems we also can conclude that the pairwise $\bar{K}N$ interaction plays a major role in the formation of the kaonic bound state and the effective chiral $\bar{K}N$ interaction gives relatively modest binding for the K^-ppm, K^-ppp and K^-K^-pp clusters.

All our calculations with the effective chiral $\bar{K}N$ interaction show that the width is always larger than the bindingenergy. In some cases the width is more than twice as large as the binding energy. Only for some four-particle kaonic clusters when the input for the $\bar{K}N$ interaction is the AY potential, the binding energy is larger than the width. As a consequence, perhaps, we are facing a situation where it is hard to identify the resonances which would make the experimental observation challenging.

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References

[1] G.E. Brown. M. Rho. Chiral restoration in hot and/or dense matter // Phys. Rep. – 1996. – Vol. 269. – P.333.

[2] C.H. Lee. Kaon condensation in dense stellar matter// Phys. Rep. – 1996. – Vol. 275. – P. 255.

[3] O.L. Berman. R.Ya. Kezerashvili. G.V. Kolmakov. and Yu.E. Lozovik. Turbulence in a Bose-Einstein condensate of dipolar excitons in coupled quantum wells // Phys. Rev. B. – 2012. – Vol. 86. – P.045108.

[4] O. L. Berman. R. Ya. Kezerashvili. K. Ziegler. Superfluidity and collective properties of excitonicpolaritons in gapped graphene in a microcavity // Phys. Rev. B. – 2012. – Vol.86. – P. 235404.

[5] Y. Akaishi and T. Yamazaki.Nuclear K^- bound states in light nuclei // Phys. Rev. C. – 2002. – Vol. 65. – P.044005.

[6] T. Yamazaki and Y. Akaishi. (K^-,π^-) production of nuclear $\bar{K}$ bound states in proton-rich systems via $\Lambda*$ doorways // Phys. Lett. B. – 2002. – Vol. 535. – P. 70.

[7] T. Yamazaki. A. Doté. and Y. Akaishi. Invariant-mass spectroscopy for condensed single- and double-$\bar{K}$ nuclear clusters to be formed as residues in relativistic heavy-ion collisions // Phys. Lett. B. – 2004. – Vol. 587. – P. 167.

[8] A. Doté. H. Horiuchi. Y. Akaishi. and T. Yamazaki. Invariant-mass spectroscopy for condensed single- and double-$\bar{K}$ nuclear clusters to be formed as residues in relativistic heavy-ion collisions // Phys. Lett. B. – 2004. – Vol. 590. – P. 51.

[9] A. Doté. H. Horiuchi. Y. Akaishi. and T. Yamazaki. Kaonic nuclei studied based on a new framework of antisymmetrized molecular dynamic // Phys. Rev. C. – 2004. – Vol. 70. – P. 044313.

[10] T. Yamazaki and Y. Akaishi. Basic K- nuclear cluster, K^-pp, and its enhanced formation in the $p+p\rightarrow K^+X$ reaction // Phys. Rev. C. – 2007. – Vol. 76. – P. 045201.

[11] A. Doté. H. Horiuchi. Y. Akaishi. and T. Yamazaki.The Study of Deeply Bound Kaonic Nuclei with Antisymmetrized Molecular Dynamics// Prog. Theor. Phys. Suppl. – 2002. – Vol. 146. – P. 508.

[12] A. Doté and W. Weise.Study of Light Kaonic Nuclei with a Chiral SU(3)-Based KN Interaction // Prog. Theor. Phys. Suppl. – 2007. – Vol. 168. – P. 593.

[13] A. Doté. T. Hyodo. and W. Weise. K^-pp system with chiral SU(3) effective interaction // Nucl. Phys. A 804. – 2008. – Vol. – P. 197.

[14] A. Doté. T. Hyodo. and W. Weise. Variational calculation of the ppK$^-$ system based on chiral SU(3) dynamics // Phys. Rev. C. – 2009. – Vol. 79. – P. 014003.

[15] S. Wycech and A. M. Green. Variational calculations for K-–few-nucleon systems // Phys. Rev. C. – 2009. – Vol. 79. – P. 014001.

[16] N. V. Shevchenko. A. Gal. J. Mareš. Faddeev Calculation of a K^-pp Quasibound State // Phys. Rev. Lett. – 2007. – Vol. 98. – P. 082301.

[17] N. V. Shevchenko. A. Gal. and J. Mareš. and J. Révai. K–NN quasibound state and the K–N interaction: Coupled-channels Faddeev calculations of the K–NN–$\pi\Sigma$N system // Phys. Rev. C. – 2007. – Vol. 76. – P.044004.

[18] Y. Ikeda and T. Sato. Strange dibaryon resonance in the K–NN-πYN system // Phys. Rev. C. – 2007. – Vol. 76. – P. 035203.

[19] Y. Ikeda and T. Sato. Resonance energy of the K–NN$-\pi$YN system // Phys. Rev. C. – 2009. – Vol. 79. – P. 035201.

[20] A. Martnez Torres. K. P. Khemchandani. and E. Oset. Solution to Faddeev equations with two-body experimental amplitudes as input and application to JP=1/2+, S=0 baryon resonances // Phys. Rev. C. – 2009. – Vol. 79. – P. 065207.

[21] Y. Ikeda. H. Kamano. and T. Sato. Energy Dependence of $\bar{K}$N Interactions and Resonance Pole of Strange Dibaryons // Prog. Theor. Phys. – 2010. – Vol. 124. – P. 533.

[22] M. Bayar. J. Yamagata-Sekihara. and E. Oset. K–NN system with chiral dynamics // Phys. Rev. C. – 2011. – Vol. 84. – P. 015209.

[23] E. Oset. D. Jido. T. Sekihara. A. Martnez Torres. K.P. Khemchandani. M. Bayar and J. Yamagata-Sekihara. A new perspective on the Faddeev equations and the system from chiral dynamics and unitarity in coupledK–NN channels // Nucl. Phys. A. – 2012. – Vol. 881. – P. 127.

[24] M. Bayar and E. Oset. Improved fixed center approximation of the Faddeev equations for the the K–NN system with S=0 // Nucl. Phys. A. – 2012. – Vol. 883. – P.57.

[25] M. Bayar and E. Oset. The K–NN system revisited including absorption // Nucl. Phys. A. – 2013. – Vol. 914. – P. 349.

[26] . S. Maeda. Y. Akaishi. and T. Yamazaki.Strong binding and shrinkage of single and double K nuclear systems (K^-pp, K^-ppn, K^-K^-p and K^-K^-pp) predicted by Faddeev-Yakubovsky calculations // Proc. Jpn. Acad. Ser. B. – 2013. – Vol. 89. – P. 418.

[27] J. Révai and N. V. Shevchenko. Faddeev calculations of the K^-NN system with a chirally motivated K^-N interaction. II. The K^-pp quasibound state // Phys. Rev. C. – 2014. – Vol. 90. – P. 034004.

[28] R.Ya. Kezerashvili. Sh.M. Tsiklauri. I.N. Filikhin. V. M. Suslov. and B. Vlahovic.Benchmark for a quasi-bound state of the K^-ppsystem // arXiv. – 2015.– P.1508.07638 [nucl-th].

[29] N. Barnea. A. Gal. and E. Z. Liverts. Three-body calculations for the K^-pp system within potential models // Phys. Lett. B. – 2012. – Vol. 712. – P. 132.

[30] . R. Ya. Kezerashvili and Sh. M. Tsiklauri. Investigation of the structure of the few body kaonic nuclei using the method of hyperspherical functions in momentum space // EPJ Web Conf. – 2014. – Vol. 81. – P.02022.

[31] M. Agnello. G. Beer. L. Benussi et al.Evidence for a Kaon-Bound State K−pp Produced in K− Absorption Reactions at Rest // Phys. Rev. Lett. – 2005. – Vol. 94. – P.212303.

[32] -PARC E15 Collaboration: T. Hashimoto. S. Ajimura. G. Beer. et al. Search for the deeply bound K–pp state from the semi-inclusive forward-neutron spectrum in the in-flight K– reaction on helium-3 // Prog. Theor. Exp. Phys. – 2015. – P. 061D01.

[33] J-PARC E27 Collaboration: Y. Ichikawa. et al. Experiment to Search for a Nuclear Kaon Bound State K−pp // Prog. Theor. Exp. Phys. – 2014. – P. 101D03; Y. Ichikawa. et al. Observation of the “K− pp”-like structure in the d(π+, K+) reaction at 1.69 GeV/c // Prog. Theor. Exp. Phys. – 2015. – P. 021D01.

[34] HADES Collaboration: G. Agakishiev. et al. Partial Wave Analysis of the Reaction p(3.5GeV)+p->pK+Λ to Search for the "ppK−" Bound State // Phys. Lett. B. – 2015. – Vol. 742. – P. 242.

[35] T. Yamazaki. et al. Indication of a Deeply Bound and Compact K−pp State Formed in the pp→pΛK+ Reaction at 2.85 GeV // Phys. Rev. Lett. – 2010. – Vol.104. – P. 132502.

[36] T. Hyodoand W. Weise. Effective K−N interaction based on chiral SU(3) dynamics // Phys. Rev. C. – 2008. – Vol. 77. – P. 035204.

[37] W. Weise. Antikaon Interactions with Nucleons and Nuclei // Nucl. Phys. A. – 2010. – Vol. 835. – P. 51.

[38] D. Jido and Y. Kanada-Enyo. KK-N molecule state with I=1/2 and JP=1/2+ studied with a three-body calculation // Phys. Rev C. – 2008. – Vol. 78. – P. 035203.

[39] Y. Kanada-Enyo. and D. Jido. K-K-N molecular state in a three-body calculation // Phys. Rev. C. – 2008. – Vol. 78. – P. 025212.

[40] T. Hyodo. S. I. Nam. D. Jido. and A. Hosaka. Flavor SU(3) breaking effects in the chiral unitary model for meson-baryon scatterings // Phys. Rev. C. – 2003. – Vol.68. – P. 018201; Detailed Analysis of the Chiral Unitary Model for Meson-BaryonScattering with Flavor SU(3) Breaking Effects // Prog. Theor. Phys. – 2004. – Vol. 112.– P.73-97.

[41] R.B. Wiringa. R. A. Smith. and T.L. Ainsworth. Nucleon-nucleon potentials with and without Δ(1232) degrees of freedom // Phys. Rev. C. – 1984. – Vol. 29. – P.1207.

[42] R. B. Wiringa. V. G. J. Stoks. and R. Schiavilla. Accurate nucleon-nucleon potential with charge-independence breaking // Phys. Rev. C. – 1995. – Vol. 51. – P. 38.

[43] R. A. Malfliet and J. A. Tjon. Solution of the Faddeev equations for the triton problem using local two-particle interactions // Nucl. Phys. A. – 1969. – Vol.127. – P. 161.

[44] R. Tamagaki. Superfluid State in Neutron Star Matter. IGeneralizedBogoliubov Transformation and Existence of 3P2 Gap at High Density // Prog. Theor. Phys. – 1970. – Vol. 44. – P. 905.

[45] D. R. Thompson. M. LeMere. Y. C. Tang. Systematic investigation of scattering problems with the resonating-group method // Nucl. Phys. A. – 1977. – Vol.286. – P. 53.

[46] R. I. Jibuti and N. B. Krupennikova. Method of hyperspherical functions for few-body systems. (in Russian) Metsniereba. Tbilisi.– 1984.

[47] J. Avery. Hyperspherical Harmonics. Kluwer Academic. Dordrecht.– 1989.

[48] R. I. Jibuti. K. V. Shitikova. Method of hyperspherical functions in atomic and nuclear physics. (in Russian) Energoatomizdat. Moscow.– 1993.

[49] R. I. Jibuti. N. B. Krupennikova. V. Yu. Tomchinsky. Hyperspherical basis for the continuum spectrum// Nucl. Phys. A. – 1977. – Vol. 276. – P. 421.

[50] R. I. Jibuti. R. Ya. Kezerashvili. Double-charge-exchange reactions of π-mesons on three- and four-particle nuclei // Nucl. Phys. A. – 1985. – Vol.437. – P. 687.

[51] J. Raynal and J. Révai.Transformation coefficients in the hyperspherical approach to the three-body problem // NuovoCimento. – 1970. – Vol.68A. – P. 612.

[52] A. Gal. Recent studies of kaonic atoms and nuclear clusters // Nucl. Phys. A. – 2013. – Vol.914. – P.270.

[53] A. Doté. T. Inoue. and T. Myo.Application of a coupled-channel complex scaling method with Feshbach projection to the K–pp system // Prog. Theor. Exp. Phys.– 2015.– P. 043D02.

[54] A. Doté. T. Inoue. and T. Myo.Essential bar K cluster "K?pp" studied with a coupled-channel Complex Scaling Method + Feshbach method // J. Phys: Conf. Ser. – 2014. – Vol. 569. – P. 012084.

[55] A. Martnez Torres. K. – P.Khemchandani. and E. Oset // Phys. Rev. C. – 2009. – Vol. 79. – P. 065207.

[56] A. Martnez Torres. K. – P.Khemchandani. D. Jido. Y. Kanada-Enyo. and E. Oset // Nucl. Phys. A. – 2013. – Vol. 914. – P. 280.

[57] A. Martnez Torres and D. Jido. Solution to Faddeev equations with two-body experimental amplitudes as input and application to JP=1/2+, S=0 baryon resonance // Phys. Rev. C. – 2010. – Vol. 82. – P.038202.

[58] S. R. Beane. et al.K+K+ scattering length from lattice QCD // Phys. Rev. D. – 2008. – Vol. 77. – P.094507.

[59] A. Martnez Torres. D. Jido. and Y. Kanada-Enyo. Theoretical study of the $KK\bar{K}$ system and dynamical generation of the K(1460) resonance // Phys. Rev. C. – 2011. – Vol. 83. – P. 065205.

[60] Y. Akaishi. K. S. Myint. and T. Yamazaki. Kaonic nuclear systems KbarN and KbarNN as decaying states // Proc. Jpn. Acad. Ser. B. – 2008. – Vol. 84. – P. 2

[61] E. Hiyama. Y. Funaki. N. Kaiser. and W. Weise.Alpha-clustered hypernuclei and chiral SU(3) dynamics // Prog. Theor. Exp. Phys. – 2014. – P. 013D01.